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ABOUT THIS GUIDE

Edgenuity supplies teachers with a Teacher’s Guide to assist them in helping students succeed in every course. The Teacher’s Guide summarizes both the content that students learn in Geometry based on the Louisiana Student Standards for Mathematics and the eight Standards for Mathematical Practice, in which students must be engaged for all of this content. The guide organizes the focus standards into units and the learning goals into lessons. Discussion questions for each unit are included, paired with tips for effective discussions to support teachers in hosting them. Online lessons begin with a warm-up, dive into rich instruction, recap what students have learned in a summary, allow students to practice skills, and finally assess students in a quiz. When the structure of the online lessons are used as designed and combined with engaging non-routine problems and deep discussions with the questions provided, teachers are given everything they need to help their students succeed in a blended learning environment.

While technology has changed how content is delivered, it has not removed the student’s need for individualized instruction, remediation or challenge—support that only a teacher can provide. That’s why each Teacher’s Guide comes with specific instructions for how to use Edgenuity’s innovative course customization toolset. This allows permissioned educators and district administrators to create truly customized courses that can meet the demands of the most rigorous classroom or provide targeted assistance for struggling students.

Finally, the Teacher’s Guide provides helpful resources, including lists of vocabulary and key interactives tools that appear in online lessons.

SUMMARY OF GEOMETRY MATHEMATICS CONTENT

Edgenuity Geometry strictly adheres to the content specified by the Common Core State Standards in conjunction with the Louisiana Student Standards for Mathematics. Students build on the geometric concepts they learned in grades six to eight by “explor[ing] more complex geometric situations and deepen[ing] their explanations of geometric relationships, moving towards formal mathematical arguments.

Students define congruence in terms of rigid motions, use similarity to define trigonometric ratios, and model with geometry. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.”
descriptions below, from the official Common Core Traditional Pathway for Geometry¹, summarize the areas of instruction for this course.

**Critical Area 1**

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions—translations, reflections, and rotations—and have used these to develop notions about what it means for two objects to be congruent. In this course, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

**Critical Area 2**

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention paid to special right triangles and the Pythagorean theorem. Students relate trigonometric ratios of similar triangles and the acute angles of a right triangle and write ratios for sine, cosine, and tangent. Students use these trigonometric ratios to solve for missing side lengths and angle measures of right triangles.

**Critical Area 3**

Students’ experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area, and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross sections and the result of rotating a two-dimensional object about a line.

**Critical Area 4**

Building on their work with the Pythagorean theorem in eighth grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines. Students also make algebraic connections as they use coordinate algebra to write equations of parabolas and circles.

**Critical Area 5**

Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. Students give informal arguments for the formulas of the circumference of a circle, area of a circle, and area of a sector. In the Cartesian coordinate system, students use the

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distance formula to write the equation of a circle when given the radius and the coordinates of its center.

**Critical Area 6**

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

**Standards for Mathematical Practice in Edgenuity Geometry**

The Standards for Mathematical Practice complement the content standards so students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout elementary, middle, and high school. These standards are the same at all grades from kindergarten to 12th grade, but the ways in which students practice them are unique to each course.

1. **Make sense of problems and persevere in solving them**

   Students in Edgenuity Geometry make sense of problems and persevere in solving them when they work through a geometric proof, identifying which theorems, propositions, and definitions may be used to prove a statement, and succeed in completing the proof. At this level, students learn that they may need to take several intermediate steps before they are able to make a critical assumption about what they are trying to prove. Students may check their reasoning by asking themselves “Is there another way to prove this theorem?” or “Does my reasoning flow logically?” Edgenuity Geometry students also make sense of problems and persevere in solving them when they identify which theorems they can use to solve an applied problem.

2. **Reason abstractly and quantitatively**

   Edgenuity Geometry students reason abstractly when they explore and make conjectures about geometric relationships. They reason quantitatively when they show these conjectures work for a specific problem. Students use geometric shapes, their measures, and their properties to describe a variety of real-world objects; they use relationships within the geometric descriptions to solve problems. For example, students contextualize by creating triangles for real-world scenarios, then decontextualize by solving for missing side lengths and angle measures of the triangles they use to model these real-world scenarios.

3. **Construct viable arguments and critique the reasoning of others**

   Constructing viable arguments is a major component of geometry. Students learn deductive reasoning and apply it throughout the course as they write proofs in multiple formats, including two-column, flowchart, and paragraph. They prove theorems about lines, angles, triangles, and other shapes. Students also construct viable arguments by deriving formulas and equations for important geometric
objects, justifying their steps as they do so. At this level, students also explain why a concept works and challenge or defend another student’s work.

4. **Model with Mathematics**

In Edgenuity Geometry, students use geometric shapes, their measures, and their properties to describe real-world objects in two and three dimensions. Special attention is paid to solving right triangles in applied problems. Edgenuity Geometry students also apply geometric methods to solve design problems. They apply concepts of density based on area and volume in modeling situations. As students move through a modeling problem, they create the model, solve problems using the model, and interpret their answers in the context of the object(s) they are modeling.

5. **Use Appropriate Tools Strategically**

Students largely use appropriate tools strategically when they make formal geometric constructions. Students consider available tools such as compasses, straight edges, protractors, and interactive tools when making these constructions. Students may also choose tools such as graphing paper or tracing paper to represent rigid and nonrigid transformations of figures in the coordinate plane.

6. **Attend to Precision**

Edgenuity Geometry students learn to communicate their ideas effectively using clear and accurate mathematical language. Students build this skill by first learning precise geometric definitions and then advance their skills by learning how to use these definitions correctly when writing proofs. Writing proofs requires clear and precise mathematical language and arguments. Students also hone this skill by knowing which theorems can be applied to solve a specific problem. Students continue the practice of attending to precision when they make exact geometric constructions, such as bisecting angles and segments. Finally, students in geometry must be precise as they identify and use corresponding parts of figures to reason about and create similar and congruent figures.

7. **Look for and Make Use of Structure**

Students are given ample opportunities to develop this practice in multiple domains of Edgenuity Geometry. Students may use the structure of a geometric figure to extend its components or draw additional pieces of it to reveal relationships within the figure that were not previously visible. For example, students may use a vertex of a right triangle and the side opposite it to sketch parallel lines with the hypotenuse cutting through these lines as a transversal. Then they use what they know about alternate interior angles to help prove that the sum of the interior measures of a triangle is equal to 180°. Geometry students look for and make use of structure when they use a geometric object to describe a real-world object. Students look for and make use of structure when conjecturing about and describing relationships among geometric objects. As a major focus in high school geometry, students also look for and make use of structure when they plan how to execute a proof using given facts and information. This includes analyzing a geometric figure to decide what theorem they can use to reason and draw conclusions about geometric figures.
8. Look for and express regularity in repeated reasoning

Edgenuity Geometry students look for and express regularity in repeated reasoning. Through students’ supporting work with transformations, they can explain how the criteria for triangle congruence follow from the definition of congruence in terms of rigid motions. Additionally, students have many opportunities to explore properties of similar figures. Through their investigations, students are able to use this repeated reasoning to discover how similarity leads to definitions of trigonometric ratios for acute angles in right triangles. Students may ask themselves “What approach will make my work more efficient?” when choosing a trigonometric function to solve for side lengths and angles of right triangles using the given information. Finally, students’ extensive practice learning and applying theorems about lines, angles, triangles, and other shapes allows them to look for similarities and patterns in relationships they have not proven before.

Focus in Edgenuity Geometry

Unit 1: Foundations of Euclidean Geometry

Estimated Unit Time: approx. 13.5 Class Periods

In the unit Foundations of Euclidean Geometry, students learn the skills they need to complete complex geometric proofs throughout the course. Students learn to describe undefined terms of Euclidean geometry, such as point and line, and then apply these ideas to define geometric terms. They then apply postulates about lines and angles to determine measures in figures. They build on their knowledge from eighth grade, where they used informal arguments to establish facts about angles in triangles and about the angles created when parallel lines are cut by a transversal. They prove statements about linear pairs, vertical angles, complementary angles, and supplementary angles. Through the proofs, students construct arguments and justify their conclusions (MP3). They also communicate precisely and try to use clear mathematical language when presenting their reasoning (MP6). Students develop skills using several interactive tools, including an interactive two-column proof tool that allows them to drag and drop statements and reasons to complete proofs and interactive compass and straightedge tools that allow students to complete constructions (MP5).

Unit 1 Focus Standards

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

In the following table, green highlights indicate major work of the grade, blue highlights indicate supporting work, and yellow highlights indicate additional work.
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<thead>
<tr>
<th>Standard Text</th>
<th>Standard ID</th>
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<tbody>
<tr>
<td>Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</td>
<td>GM:G-CO.A.1</td>
</tr>
<tr>
<td>Prove and apply theorems about lines and angles. <em>Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.</em></td>
<td>GM:G-CO.C.9</td>
</tr>
<tr>
<td>Make formal geometric constructions with a variety of tools and methods, e.g., compass and straightedge, string, reflective devices, paper folding, or dynamic geometric software. Examples: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</td>
<td>GM:G-CO.D.12</td>
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**Unit 1 Pacing Guide**

<table>
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<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
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| **Euclidean Geometry**      | • Identify and name undefined terms of point, line, plane, and distance along a line.  
• Analyze descriptions and diagrams that illustrate basic postulates about points, lines, and planes.                                             | GM:G-CO.A.1        | 1.5            |
| **Defining Terms**          | • Use undefined terms to precisely define parallel lines, perpendicular lines, ray, angle, arc, circle, and line segment.  
• Identify and name a pair of parallel lines, a pair of perpendicular lines, a ray, an angle, an arc, a circle, and a line segment.  | GM:G-CO.A.1        | 1.5            |
| **Measuring Length and Angles** | • Identify a midpoint or bisector of a line segment or angles.  
• Apply the ruler postulate and segment addition postulate to calculate the lengths of line segments.  
• Apply the protractor postulate and angle addition postulate to calculate angle measures.                                      | GM:G-CO.A.1        | 1.5            |
| **Introduction to Proof**   | • Complete the steps to prove algebraic and geometric statements.  
• Identify proof formats, the essential parts of a proof, and the assumptions that can be made from a given drawing.                                        | GM:G-CO.C.9        | 2               |
## Lesson Objectives

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<th>Lesson</th>
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<th>Standards</th>
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| Linear Pairs and Vertical Angles            | - Calculate angle measures by using definitions and theorems about linear pairs and vertical angles.  
- Identify linear pairs and vertical angles from given diagrams.  
- Complete the steps to prove statements using linear pairs and vertical angles. | GM:G-CO.C.9     | 2              |
| Complementary and Supplementary Angles      | - Identify complementary angles and supplementary angles from given diagrams.  
- Solve problems involving measures of complementary and supplementary angles.  
- Complete the steps to prove statements using complementary angles and supplementary angles. | GM:G-CO.C.9     | 2              |
| Performance Task: Constructions             |                                                                                                                                                                                                 | GM:G-CO.D.12    | 2              |
| Unit Test                                   |                                                                                                                                                                                                 |                | 1              |

### Discussion Questions & Answers

1. What do all proofs have in common? Name and describe three types of proofs.
   - All proofs contain given information and a statement to be proven. You use deductive reasoning to create an argument with justification of steps using theorems, postulates, and definitions. Then you arrive at a conclusion.
   - One type of proof is a two-column proof. It contains statements and reasons in columns. Another type is a paragraph proof, in which statements and reasons are written in words. A third type is a flowchart proof, which uses a diagram to show the steps of a proof.

2. Imagine two lines intersect. How can the properties of linear pairs and vertical angles help to determine the angle measures created by the intersecting lines? Explain.
   - If you know the measure of one angle in a linear pair, you can find the measure of the other because the sum of the measure of the two angles is 180°. If you know the measure of one angle in a pair of vertical angles, you can find the measure of the other because they are congruent.

3. What criteria does a mathematical term need to meet to be defined? What terms are undefined in geometry?
   - Mathematical definitions must
     - be precise and clearly represent the concept without ambiguity
     - have sufficient information to describe any variation of the concept
include only the necessary information required to encompass the characteristics of the concept

b. The terms point, line, plane, distance along a line, and distance along an arc cannot be precisely defined in geometry.

Common Misconceptions

- **Angle Measurement**
  - Students may assume that if the side lengths of an angle are extended, the angle measure is increased.
  - Students may assume that any two adjacent angles add up to either 90° or 180°.
  - Students confuse measures of supplementary and complementary angles.

- **Proof**
  - Students may assume that a statement only has to hold once or a few times to be considered true.
  - Students may use “prove” for the last reason in a two-column proof because the first reason in the proof is “given.”
  - Students may assume what they are trying to prove.

- **Congruence**
  - Students may assume that two segments or angles that look the same are congruent, even if they are not marked.

- **Language**
  - Students may develop an image of a vocabulary word without a concept definition, leading to false assumptions. For example, when students picture supplementary angles, they may picture that they are always adjacent. Therefore, they would not state that a pair of nonadjacent angles that add up to 180° are supplementary angles.

- **Using a protractor**
  - Students may incorrectly place a protractor in the horizontal position, regardless of the orientation of the angle being measured.

Classroom Challenge

1. Draw a pair of intersecting lines. Name the angle relationship that is formed between each pair of angles. How many pairs of angles are made?
2. Draw three lines that intersect at a single point such that the diagram includes at least one pair of complementary and supplementary angles. Write at least three equations relating the measures of the angles in the diagram to the definitions of complementary and supplementary.
Possible solution pathway:

1. 

There will be six pairs of angles.

\(\angle ABC\) and \(\angle CBE\) are linear pairs. \(\angle ABC\) and \(\angle EBD\) are vertical angles.

\(\angle ABC\) and \(\angle ABD\) are linear pairs. \(\angle CBE\) and \(\angle EBD\) are linear pairs.

\(\angle CBE\) and \(\angle ABD\) are vertical angles. \(\angle EBD\) and \(\angle DBA\) are linear pairs.

2. In order to include a pair of complementary angles, two of the three lines must be perpendicular.

\[m\angle BEF + m\angle CEA = 180^\circ\]

\[m\angle BED + m\angle DEC = 90^\circ\]

\[m\angle FEG + m\angle GEA = 90^\circ\]

Teacher notes:
For struggling students, ask them to write the different types of angle pair relationships they remember from the instruction. Then ask them to identify any angles they see in the diagrams that satisfy those definitions. Emphasize that there are only two types of pairs of angles in each part of the problem.
For advanced students, ask them how changing the intersecting lines in the first part of the problem to a pair of perpendicular lines changes their pairings. Then ask them how changing the third line to a ray with its endpoint at the intersection in the second part of the problem changes the possible equations in both type and number possible.

UNIT 2: GEOMETRIC TRANSFORMATIONS

Estimated Unit Time: approx. 13.5 Class Periods

In the unit Geometric Transformations, students learn about different types of transformations. This supports the major work of showing triangle congruence and triangle similarity in terms of rigid and nonrigid motions respectively that they learn in later units. They build on their knowledge of transformations from eighth grade. Students learn to identify different types of transformations and classify them as isometric or not isometric by using constructions to build definitions. They use technological tools appropriately and strategically to deepen their understanding of the transformations (MP5). Students describe mappings using symbols that communicate relationships between corresponding parts of the pre-image and image and write a rule that describes a given transformation (MP6). Students learn to combine reflections, rotations, and translations and determine the sequence of transformations that will carry a given figure onto another figure. They also learn how to draw transformations. Finally, they identify rotational and reflectional symmetry in geometric figures.

Unit 2 Focus Standards

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

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<tbody>
<tr>
<td>Represent transformations in the plane using, e.g., transparencies, tracing</td>
<td>GM:G-CO.A.2</td>
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<td>paper, or geometry software; describe transformations as functions that</td>
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<td>take points in the plane as inputs and give other points as outputs.</td>
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<tr>
<td>Compare transformations that preserve distance and angle to those that do</td>
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<td>not (e.g., translation versus horizontal stretch).</td>
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<tr>
<td>Given a rectangle, parallelogram, trapezoid, or regular polygon, describe</td>
<td>GM:G-CO.A.3</td>
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<tr>
<td>the rotations and reflections that carry it onto itself.</td>
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<tr>
<td>Develop definitions of rotations, reflections, and translations in terms of</td>
<td>GM:G-CO.A.4</td>
</tr>
<tr>
<td>angles, circles, perpendicular lines, parallel lines, and line segments.</td>
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</tbody>
</table>
### Standard Text

Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

### Standard ID

| GM:G-CO.A.5 |

### Unit 2 Pacing Guide

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
</table>
| **Introduction to Transformations** | • Determine if a transformation is isometric and identify corresponding parts of the pre-image and image.  
• Identify the type of transformation given a pre-image and an image. | GM:G-CO.A.2 | 1.5 |
| **Reflections** | • Develop the definition of a reflection using constructions.  
• Describe the properties of and write rules for reflections.  
• Determine the image or pre-image of a figure after a given reflection. | GM:G-CO.A.2  
GM:G-CO.A.4  
GM:G-CO.A.5 | 2 |
| **Translations** | • Develop the definition of a translation using constructions.  
• Write the rule that describes a given translation.  
• Determine the image or pre-image of a figure after a given translation. | GM:G-CO.A.4  
GM:G-CO.A.2  
GM:G-CO.A.5 | 2 |
| **Rotations** | • Develop the definition of a rotation using constructions.  
• Describe the properties of and write rules for rotations.  
• Determine the image or pre-image of a figure after a given rotation. | GM:G-CO.A.4  
GM:G-CO.A.2  
GM:G-CO.A.5 | 2 |
| **Compositions** | • Determine the rule that describes a given composition of transformations.  
• Determine the image of a figure after a given composition of transformations. | GM:G-CO.A.5  
GM:G-CO.A.2 | 2 |
| **Symmetry** | • Identify reflectional symmetry in geometric figures and the number of lines of symmetry.  
• Identify rotational symmetry and its order in geometric figures. | GM:G-CO.A.3 | 1.5 |
Lesson | Objectives | Standards | Number of Days
--- | --- | --- | ---
Drawing Transformations | • Draw the translation of a given figure using graph paper or tracing paper.  
• Draw the reflection of a given figure using graph paper or tracing paper.  
• Draw the rotation of a given figure using graph paper or tracing paper.  
• Determine the sequence of transformations that will carry a given figure onto another figure. | GM:G-CO.A.2  
GM:G-CO.A.5 | 1.5

Unit Test |  |  | 1

Discussion Questions & Answers
1. What do all rigid transformations have in common? How does this help us compare the area and perimeter of an image and its pre-image?
   a. All rigid transformations preserve side length and angle measure. This means that the area and perimeter of an image will be the same as the area and perimeter of its pre-image.

2. The commutative property tells us that the order in which two numbers are added or multiplied does not matter. Is a composition of transformations commutative? Describe a sequence of transformations where the order does not matter. Describe a sequence of transformations where the order does matter.
   a. A composition of transformations is not commutative so it should not be assumed that a composition performed in the opposite order will produce the same image.
      i. Consider an isosceles triangle with vertices A(0, 1), B(-4, 1), and C(-2, 5).
         Performing the composition $R_{180^\circ} \circ r_y(x, y)$ ends in vertices of $A''(0, -1), B''(-4, -1),$ and $C''(-2, -5).$ Performing the composition $r_y(x, y) \circ R_{180^\circ}(x, y)$ ends in vertices of $A''(0, -1), B''(-4, -1),$ and $C''(-2, -5).$
      ii. Consider an isosceles triangle with vertices A(0, 1), B(-4, 1), and C(-2, 5).
         Performing the composition $T_{2,4} \circ r_y(x, y)$ ends in vertices of $A''(3, 4), B''(3, 0),$ and $C''(7, 2).$ Performing the composition $r_y(x, y) \circ T_{2,4}(x, y)$ ends in vertices of $A''(5, 2), B''(5, -2),$ and $C''(0, 9).$

3. How does rotational symmetry affect the results of rigid transformations?
   a. Consider a series of compositions includes a rotation or other combination of transformations mimicking a rotation. In these compositions, it could appear that the orientation has not changed even though the composition includes a transformation that does not preserve orientation. For figures with rotational symmetry, extra care should be taken when dealing with corresponding vertices.
Consider rectangle $A(1, 2), B(1, 4), C(5, 2), \text{ and } D(5, 4)$. A reflection over the x-axis does not preserve orientation. The result of performing $r_{x-axis} \circ r_{y-axis}$ is $A'(-1, -2), B'(-1, -4), C'(-5, -2), \text{ and } D'(-5, -4)$. The image has vertical sides that are 2 units and horizontal sides that are 2 units. This matches the properties of the pre-image. However, corresponding vertices are not in the same location. A has moved from the bottom left-hand corner to the top right-hand corner. This optical illusion can be explained because the figure has rotational symmetry.

Common Misconceptions

- **Rotations**
  - Students may think the center of rotation is limited to the origin or a central point within the shape rather than understanding that the center of a rotation can be any point.

- **Reflections**
  - Students may think that lines of reflection are limited to horizontal or vertical lines rather than understanding that any line can be used for a line of reflection.

- **Lines of symmetry**
  - Students may believe that because a line divides a figure in half that the line is a line of symmetry. A classic example of this is the diagonal of a rectangle that divides the shape in half but is not a line of symmetry.

Classroom Challenge

Start with an irregular polygon contained completely in the second quadrant. Duplicate this pre-image two additional times on separate coordinate grids.

1. On the first coordinate grid, perform a translation. Write a mapping rule for the translation and give a verbal description of the translation. Write a description of the translation.
2. On the second coordinate grid, perform a rotation. Write a mapping rule for the rotation and give a verbal description of the rotation. Write a description of the rotation.
3. On the third coordinate grid, perform a reflection. Write a mapping rule for the reflection and give a verbal description of the reflection. Write a description of the reflection.
4. For each transformation, describe at least one feature of the image that is the same as the pre-image and one feature of the image that is different from the pre-image.
Possible solution pathway:

A graph of a scalene triangle satisfies in the criteria of irregular polygon. We can use the vertices (-2, 1), (-3, 2), and (-1, 5) to create the pre-image.

1. A translation of $T_{-5,-10}(x, y) \rightarrow (x - 5, y - 10)$ creates the image in the graph. The polygon shifted 5 units to the left and 10 units down.

2. Consider $R_{0,270^\circ}(x, y) \rightarrow (y, -x)$ that creates the image in the graph.

3. Consider $r_{x=1}(x, y) = (-x + 2, y)$ that creates the image in the graph.
4. For the translation, the orientation of the triangle stayed the same, but the sign of the y-coordinates of the vertices changed from positive to negative. For the rotation, the lengths of the sides of the triangle stayed the same, but the orientation of the triangle changed. For the reflection, the measure of the angles stayed the same, but the orientation of the triangle changed.

Teacher notes:

For students who struggle, suggest they write the description of the transformations and then apply them. When looking for similarities and differences, remind students of the defining features of a polygon as well as their understanding of points in various quadrants on the coordinate plane.

For advanced students, give more specific criteria. Ask them to perform transformations so that the pre-image is completely contained in the second quadrant and the image is completely contained in the fourth quadrant. These students could also create a composition of transformations that maps the image and pre-images given in the problem.

**UNIT 3: ANGLES AND LINES**

*Estimated Unit Time: approx. 11 Class Periods*

In the unit Angles and Lines, students apply understanding of parallel and perpendicular lines to prove relationships between angles and write equations of lines. Students identify key features of angles created when a transversal crosses parallel lines and justifies these features through proofs (MP3). Students build on their knowledge of these angles from eighth grade, when they used informal arguments. They prove the slope criterion for parallel and perpendicular lines and use the relationships discovered to solve mathematical problems (MP1).

**Unit 3 Focus Standards**

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

In the following table, green highlights indicate major work of the grade, blue highlights indicate supporting work, and yellow highlights indicate additional work.
### Standard Text

Prove and apply theorems about lines and angles. **Theorems include**: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.

Make formal geometric constructions with a variety of tools and methods, e.g., compass and straightedge, string, reflective devices, paper folding, or dynamic geometric software. Examples: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

### Standard ID

- GM:G-CO.C.9
- GM:G-CO.D.12
- GM:G-GPE.B.5

### Unit 3 Pacing Guide

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
</table>
| Parallel and Perpendicular Lines | • Construct parallel and perpendicular lines.  
• Identify parallel, perpendicular, and skew lines from three-dimensional figures.  
• Solve problems involving the distance from a point on the perpendicular bisector to both endpoints of the line segment.                                                                                                                                                                     | GM:G-CO.C.9  
GM:G-CO.D.12       | 2 |
| Lines Cut by a Transversal     | • Solve for angle measures when parallel lines are cut by a transversal.  
• Complete the steps to prove angle relationships given parallel lines cut by a transversal.                                                                                                                                                                                   | GM:G-CO.C.9       | 2 |
| Proving Lines Parallel        | • Apply theorems to determine if lines are parallel.  
• Prove lines are parallel given angle relationships.                                                                                                                                                                                                                                   | GM:G-CO.C.9       | 2 |
| Slopes of Parallel and Perpendicular Lines | • Complete the steps to prove the slope criteria for parallel and perpendicular lines using coordinate geometry.  
• Determine if two lines are parallel or perpendicular.  
• Use slope criteria to find additional points on a line parallel or perpendicular to a given line.  
• Prove the slope criteria for perpendicular lines.                                                                                                                                                                                                 | GM:G-GPE.B.5       | 2 |
Discussion Questions & Answers

1. Analyze constructions of parallel lines to determine which transformations are used. Evaluate the effect of these transformations to the proof of the relationships between the pairs of angles created when parallel lines are cut by a transversal.
   a. Construction of parallel lines uses a translation. Since translations are the movement of every point in a figure in the same distance and direction, translating a line creates a parallel line. One of the rays illustrating the direction of the translation becomes part of the transversal creating congruent corresponding angles. This congruency relationship is the foundation for the proofs of all the relationships between angles created when a pair of parallel lines is cut by a transversal.

2. When writing the equations of parallel and perpendicular lines, describe the slope and $y$-intercept for each pair of lines.
   a. In parallel lines, the slopes must be the same and the $y$-intercepts are always different since the lines never intersect. In perpendicular lines, the slopes are negative reciprocals, but the $y$-intercepts could be the same since the lines always intersect.

3. Consider the relationships between slopes of parallel and perpendicular lines to determine a time when they are useful for proving properties of shapes on a coordinate plane.
   a. Finding the slope of the sides in a quadrilateral can help determine if the quadrilateral can be further classified. For instance, to classify the quadrilateral as a parallelogram, the slopes of opposite sides must be equal. Furthermore, to classify a parallelogram as a rectangle, the slopes of adjacent sides must be negative reciprocals.

Common Misconceptions
- Parallel lines
  - Students may attempt to classify lines as parallel without using necessary criteria of angle relationships when the lines are intersected by a transversal or the slopes.

- Perpendicular lines
  - Students may attempt to classify lines as perpendicular without using necessary criteria of the slopes.
Students may not realize that the y-intercepts of perpendicular lines could be the same.

- Angles created when parallel lines are cut by a transversal
  - Students may believe that all pairs of angles are congruent rather than separating relationships where the pairs of angles are supplementary.

Classroom Challenge

One side of a rectangle is given in the graph. Write the equation of the line that contains the given side.

Use the characteristics of a rectangle to write the equations of lines that could contain the three additional sides.

Possible solution pathway:

The slope of the line in the graph is -1. The line contains the point (2, 3). Using point slope form:

\[ y - 3 = -1(x - 2) \]
\[ y = -x + 5 \]

The equation for one side of the rectangle perpendicular to the first side includes the point (2, 3) and a slope of 1, which is the negative reciprocal of the first slope. Using point-slope form:

\[ y - 3 = 1(x - 2) \]
\[ y = x + 1 \]

The equation for the second side perpendicular to the first side includes the point (6, -1) and a slope of 1, which is the negative reciprocal of the first slope. Using point-slope form:

\[ y + 1 = 1(x - 6) \]
\[ y = x - 7 \]
The equation for a third unknown side can be many equations that include a point on both the second and third lines and having a slope of -1, which equals the slope of the first line. Using the point (-1, 0) in point-slope form:

\[ y - 0 = -1(x + 1) \]
\[ y = x - 1 \]

**Teacher notes:**

For struggling students, remind them they can count the rise and run when finding the slope of the line containing the given side. Students can also graph points on the lines containing possible additional sides using the given vertices and the slope depending on the relationship between the lines.

Advanced students can be asked to describe which equations, if any, must be the same for all answers and which equations, if any, can vary. Teachers can also challenge them to determine the equation of lines that contain a square created from the given side.

**UNIT 4: TRIANGLES**

*Estimated Unit Time: approx. 16 Class Periods*

In the unit Triangles, students begin specific work with proofs of triangle theorems (MP3). They build on their knowledge of triangles from seventh and eighth grade. Throughout this unit, they utilize interactive tools to discover and verify through experimentation statements about relationships between triangle angles and sides, sums of measures of parts of triangles, and triangle constructions (MP5 and MP8). Students prove and apply theorems about isosceles triangles. They calculate unknown measures in triangles and in parts of triangles formed by perpendicular or angle bisectors.

**Unit 4 Focus Standards**

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

In the following table, green highlights indicate major work of the grade, blue highlights indicate supporting work, and yellow highlights indicate additional work.

<table>
<thead>
<tr>
<th>Standard Text</th>
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<tbody>
<tr>
<td>Prove and apply theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</td>
<td>GM:G-CO.C.10</td>
</tr>
<tr>
<td>Standard Text</td>
<td>Standard ID</td>
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<tr>
<td>Make formal geometric constructions with a variety of tools and methods, e.g., compass and straightedge, string, reflective devices, paper folding, or dynamic geometric software. Examples: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</td>
<td>GM:G-CO.D.12</td>
</tr>
<tr>
<td>Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</td>
<td>GM:G-MG.A.1</td>
</tr>
<tr>
<td>Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.</td>
<td>GM:G-C.A.3</td>
</tr>
<tr>
<td>Prove that all circles are similar.</td>
<td>GM:G-C.A.1</td>
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</table>

**Unit 4 Pacing Guide**

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
</table>
| **Constructing and Analyzing Triangles**    | • Using a ruler, protractor, technology, or freehand drawing, construct a triangle given the measures of the three sides or angles.  
• Make generalizations about the number and types of triangles that can be drawn using the given conditions.                                                                                                                                                                                                                                                                                           | GM:G-CO.C.10 | 2              |
| **Triangle Angle Theorems**                 | • Complete the steps to prove that the sum of the measures of the interior angles of a triangle is 180°.  
• Identify and relate the interior and exterior angles of a triangle.  
• Calculate the measures of interior and exterior angles of a triangle.                                                                                                                                                                                                                                                                                     | GM:G-CO.C.10 | 2              |
| **Triangles and Their Side Lengths**        | • Construct or justify the construction of isosceles and equilateral triangles.  
• Analyze the relationships between the angles of acute, right, and obtuse triangles.  
• Determine if three given segments will satisfy the triangle inequality.  
• Determine the length or parameters for a third side of a triangle given the other two sides.                                                                                                                                                                                                                                                                  | GM:G-CO.C.10 GM:G-CO.D.12 | 2              |
| **Triangle Inequalities**                   | • Identify angle and side relationships in a triangle.  
• Identify angle and side relationships between two triangles.  
• Solve real-world problems involving relationships between angle measures and side lengths of one or two triangles.                                                                                                                                                                                                                                                      | GM:G-CO.C.10 GM:G-MG.A.1 | 2              |
Lesson | Objectives | Standards | Number of Days
--- | --- | --- | ---
Isosceles Triangles | • Complete the steps to prove the isosceles triangle theorem and its converse.  
• Identify characteristics of an isosceles triangle.  
• Solve for unknown measures of isosceles triangles. | GM:G-CO.C.10 | 2
Centroid and Orthocenter | • Complete the steps to prove that the medians of a triangle meet at a point.  
• Identify the characteristics of the centroid or orthocenter of a triangle.  
• Solve for unknown measures created by medians in a triangle. | GM:G-CO.C.10 | 2
Incenter and Circumcenter | • Construct inscribed and circumscribed circles of a triangle.  
• Identify the characteristics of the incenter or circumcenter of a triangle.  
• Solve for unknown measures created by perpendicular or angle bisectors in a triangle. | GM:G-CO.C.10  
GM:G-CO.D.12  
GM:G-C.A.3 | 2
Construct Regular Polygons | • Construct regular polygons inscribed in a circle.  
• Prove that all circles are similar. | GM:G-CO.D.13  
GM:G-C.A.1 | 1
Unit Test | | | 1

**Discussion Questions & Answers**

1. What effect does the classification of the vertex angle have on the base angles in an isosceles triangle? Describe the base angles for each of the possible types of vertex angles.
   
   a. *The vertex angle of an isosceles triangle can be either acute, right, or obtuse. No matter what type of angle the vertex angle of an isosceles triangle is, the base angles must be acute. When the vertex angle of an isosceles triangle is a right angle, the base angles must be 45°. When the vertex angle of an isosceles triangle is acute, the base angles must also be acute because the measures of the base angles of an isosceles triangle are equal and a triangle cannot have more than one obtuse angle.*

2. Explain how the construction of a circumscribed circle of a triangle is different from the construction of an inscribed circle of a triangle. Draw a conclusion about how the definition of the key point in the triangle used for the construction relates to the characteristics of the circle.
   
   a. *By definition, a circle is all the points that are equidistant from the center. The circumscribed circle of a triangle is constructed using the circumcenter, which is the point of concurrency of the perpendicular bisectors of the sides as the center of the circle. The circle using this point as its center would include all the points equidistant from the circumcenter. The circumcenter is equidistant to all the vertices of the triangle. The vertices of the triangle on the circle indicate it is circumscribed. The inscribed circle of a*
triangle is constructed using the incenter, which is the point of concurrency of the angle bisectors of the triangle as the center of the circle. Since the incenter is equidistant to the sides, the inscribed circle has a radius equal to this distance.

3. An exterior angle can be found by extending either of the sides of the triangle where it occurs. Study the relationship to show how the two possible angles are related allowing either one to be named the exterior angle of the triangle at the given vertex.
   a. The two possible exterior angles form a pair of vertical angles so they are congruent. This is why we can say that both their measures are equivalent to the sum of the measures of the two nonadjacent interior angles.

Common Misconceptions

- Using a protractor
  o Students may incorrectly place a protractor in the horizontal position, regardless of the orientation of the angle being measured.

- Angles of triangles
  o Students may label angles exterior angles that were not created by extending one side of the triangle.
  o Students may incorrectly assume that exterior angles are congruent to the adjacent angle.

- Regular polygons
  o Students may only consider polygons presented in the “normal” fashion as regular.

- Identifying parts of isosceles triangles
  o Students may incorrectly assume that the vertex is the highest point or top of the triangle rather than finding the angle that is opposite the noncongruent side as the vertex.

- Language
  o Students may develop an image of a vocabulary word without a concept definition, leading to false assumptions. For example, the centroid does not occur at the midpoint or center of the medians. Also, orthocenters and circumcenters are not always inside triangles.

Classroom Challenge

Draw an isosceles triangle and its exterior angle such that the exterior angle is congruent to its adjacent angle.

1. Considering the three interior angles and the one specified exterior angle, write three equations relating the angle measures.
2. Write three mathematical statements relating the sides of the triangle.
Possible solution pathway:

An exterior angle of a triangle and its adjacent angle are linear pairs. For them to be congruent, they must both be right angles.

1. \( m\angle DAB = m\angle ABC + m\angle CAB \); \( m\angle DAB = m\angle BAC \); \( m\angle BAC = m\angle ABC + m\angle CAB \)
2. \( BC > AB; BC > AC; AC = AB \)

Teacher notes:

For struggling students, have them start by sketching any triangle and its exterior angle. From here, visual learners will be able to see that the exterior angle and its adjacent angle form a linear pair. Remind students that the measures of a linear pair have a sum of 180°. Use this relationship to determine the nature of the final sketch.

For advanced students, ask them to complete a proof given the initial information and concluding with any of the mathematical statements in the work or classifying the triangle as a right triangle.

UNIT 5: TRIANGLE CONGRUENCE

Estimated Unit Time: approx. 13 Class Periods

In the unit Triangle Congruence, students discover and prove theorems related to triangle congruence (MP3). Students draw from their work in eighth grade and previous lessons to connect concepts of congruence and transformations. They use an interactive proof tool (MP5) and scaffolded drop-down proof activities to complete proofs of SAS, AAS, ASA, SSS, and HL congruence theorems. Students work to determine the pairs of sides and angles that can be used to prove triangle congruence using these theorems. They use the idea that corresponding parts of congruent triangles are congruent to complete proofs and to solve problems (MP1). The unit ends with an opportunity for students to apply their work with triangle congruence in a performance task.

Unit 5 Focus Standards

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.
In the following table, **green highlights** indicate major work of the grade, **blue highlights** indicate supporting work, and **yellow highlights** indicate additional work.

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<tr>
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<tbody>
<tr>
<td>Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.</td>
<td>GM:G-CO.B.6</td>
</tr>
<tr>
<td>Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.</td>
<td>GM:G-CO.B.7</td>
</tr>
<tr>
<td>Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.</td>
<td>GM:G-CO.B.8</td>
</tr>
<tr>
<td>Prove and apply theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity; SAS similarity criteria; SSS similarity criteria; ASA similarity.</td>
<td>GM:G-SRT.B.4</td>
</tr>
<tr>
<td>Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</td>
<td>GM:G-SRT.B.5</td>
</tr>
<tr>
<td>Represent transformations in the plane using, e.g., transparencies, tracing paper, or geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).</td>
<td>GM:G-CO.A.2</td>
</tr>
<tr>
<td>Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.</td>
<td>GM:G-CO.A.5</td>
</tr>
</tbody>
</table>

**Unit 5 Pacing Guide**

<table>
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<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Congruent Figures</td>
<td>• Write congruency statements for transformed figures.</td>
<td>GM:G-CO.B.7</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>• Determine if figures are congruent and, if so, identify their corresponding parts.</td>
<td>GM:G-CO.B.6</td>
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<tr>
<td></td>
<td>• Determine unknown measures of congruent figures.</td>
<td>GM:G-CO.A.2</td>
<td></td>
</tr>
<tr>
<td>Lesson</td>
<td>Objectives</td>
<td>Standards</td>
<td>Number of Days</td>
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</tr>
</tbody>
</table>
| Triangle Congruence: SAS    | • Determine the isometric transformations that would map one triangle onto another triangle given that two corresponding sides and the included angle are congruent.  
  • Identify the sides and angle that can be used to prove triangle congruency using SAS.  
  • Complete the steps to prove triangles are congruent using SAS.                                                                                     | GM:G-CO.B.8  
GM:G-CO.B.6  
GM:G-CO.A.5 | 2              |
| Triangle Congruence: ASA and AAS | • Identify the side and angles that can be used to prove triangle congruency using ASA or AAS.  
  • Complete the steps to prove triangles are congruent using ASA or AAS.  
  • Determine the isometric transformations that would map one triangle onto another triangle given that two pairs of corresponding angles and one pair of corresponding sides are congruent. | GM:G-CO.B.8  
GM:G-CO.B.6  
GM:G-SRT.B.4  
GM:G-CO.A.5 | 2              |
| Triangle Congruence: SSS and HL | • Identify the parts that can be used to prove triangle congruency using SSS or HL.  
  • Complete the steps to prove triangles are congruent using SSS or HL.  
  • Determine the isometric transformations that would map one triangle onto another triangle given that three corresponding sides are congruent. | GM:G-CO.B.8  
GM:G-CO.B.7  
GM:G-CO.B.6  
GM:G-CO.A.5 | 2              |
| Using Triangle Congruence Theorems | • Identify the triangle congruency theorem that can be used to prove two triangles congruent.  
  • Complete the steps to prove angles, segments, and triangles are congruent using triangle congruence theorems and CPCTC.                                                                 | GM:G-SRT.B.5 | 2              |
| Performance Task: Congruency Proofs |                                                                                                                                                                                                 | GM:G-SRT.B.5  
GM:G-CO.B.8 | 2              |
| Unit Test                   |                                                                                                                                                                                                 | GM:G-SRT.B.5  
GM:G-CO.B.8 | 1              |

**Discussion Questions & Answers**

1. Using corresponding parts of congruent figures are congruent (CPCFC), hypothesize how many congruency statements can be written for a polygon with \( n \) sides.
   1. All of the corresponding angles are congruent. All of the corresponding sides are congruent. If a polygon has \( n \) sides, it also has \( n \) angles. There will be \( n \) statements for the congruency of the angles and \( n \) statements for the congruency of the sides. The total number of statements is \( 2n \).
2. Compare the process of determining triangle congruency when you are given two pairs of congruent angles and one pair of congruent sides to the process of determining triangle congruency when you are given two pairs of congruent sides and one pair of congruent angles.
   a. **With two pairs of congruent angles and one pair of congruent sides, the triangles are always congruent because the triangles can be proved congruent using ASA or AAS.** When given two pairs of congruent sides and one congruent angle, the triangles are only sometimes congruent since SAS proves congruency but SSA does not.

3. Explain in your own words why the hypotenuse leg theorem (HL) becomes the exception to the rule that applies to the other congruency theorems requiring three congruent parts be shown before congruency can be stated.
   a. **Only right triangles have hypotenuses. Since the triangle is a right triangle, the sides can be found using the Pythagorean theorem. By the Pythagorean theorem, there is only one possible value for the third leg. Therefore, the remaining third sides are equal. So this is really a version of the SSS case. This means that knowing just two elements, the hypotenuse and one leg, we can say the two right triangles are congruent as an extension of proving triangle congruency using SSS.**

**Common Misconceptions**

- Identifying congruency
  - Students may assume congruency based on a visual match rather than verifying the relationship.
  - Student may assume triangles with different orientations cannot be congruent, forgetting that rotations and reflections are rigid transformations.

- Congruency statements
  - Students may forget the importance of the order of vertices when making congruency statements. The corresponding vertices need to be in the same sequence for the statement to be true.

- Proving congruency
  - Students may not recognize when they are given congruent corresponding parts that match the criteria of specific triangle congruence theorems.

**Classroom Challenge**

Start with \( \triangle ABC \), which is isosceles.

1. Perform a construction in the triangle to create two triangles that can be proven congruent using SSS. Show your construction and complete the proof using the information from the construction.
2. Perform a construction in the triangle to create two triangles that can be proven congruent using SAS. Show your construction and complete the proof using the information from the construction.
3. Perform a construction in the triangle to create two triangles that can be proved congruent using HL. Show your construction and complete the proof using the information from the construction.

**Possible solution pathway:**

1. The construction of a bisector of the side opposite the vertex of the triangle creates two triangles within the original that can be proven congruent using SSS.

   ![Diagram with construction](image)

   **Proof**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle ABC ) is isosceles with vertex at ( A ). ( \overline{AD} ) bisects ( \overline{BC} ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( BD = DC )</td>
<td>2. Definition of bisector</td>
</tr>
<tr>
<td>3. ( AB = AC )</td>
<td>3. Definition of isosceles triangles</td>
</tr>
<tr>
<td>4. ( AD = AD )</td>
<td>4. Reflexive</td>
</tr>
<tr>
<td>5. ( \triangle ADC \cong \triangle ADB )</td>
<td>5. SSS</td>
</tr>
</tbody>
</table>

2. The construction of a bisector of the vertex of the triangle creates two triangles within the original that can be proven congruent using SAS.

   ![Diagram with construction](image)

   **Proof**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{AD} )</td>
<td>( \overline{BD} )</td>
</tr>
</tbody>
</table>

© 2018 Edgenuity Inc. All Rights Reserved. May not be copied, modified, sold, or redistributed in any form without permission.
1. \( \triangle ABC \) is isosceles with vertex at A. \( \overline{AD} \) bisects \( \angle BAC \).

2. \( m \angle BAD = m \angle DAC \)

3. \( AB = AC \)

4. \( AD = AD \)

5. \( \triangle ADC \cong \triangle ADB \)

1. Given

2. Definition of bisector

3. Definition of isosceles triangles

4. Reflexive

5. SAS

The construction of the line perpendicular to the side opposite the vertex that includes the vertex creates two triangles within the original that can be proven congruent using HL.

**Proof**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle ABC ) is isosceles with vertex at A. ( \overline{AD} \perp \overline{BC} ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle ADB ) and ( \angle ADC ) are right angles.</td>
<td>2. Definition of perpendicular</td>
</tr>
<tr>
<td>3. ( AB = AC )</td>
<td>3. Definition of isosceles triangles</td>
</tr>
<tr>
<td>4. ( AD = AD )</td>
<td>4. Reflexive</td>
</tr>
<tr>
<td>5. ( \triangle ADC \cong \triangle ADB )</td>
<td>5. HL</td>
</tr>
</tbody>
</table>

**Teacher notes:**

For struggling students, start a conversation about the types of constructions that can be completed having to do with each specified element. For instance, angles can be copied and bisected. Which might be useful in creating equivalent angles? Repeat this for each of the scenarios.

For advanced students, have them describe what the three constructions have in common, the goal being to determine that the constructions are actually all the same line. These students also could explore completing a fourth proof using a construction and the AAS criteria.

**UNIT 6: SIMILARITY TRANSFORMATIONS**

*Estimated Unit Time: approx. 17 Class Periods*

In the unit Similarity Transformations, students discover properties of similarities by applying previous work on transformations from eighth grade to develop an understanding of dilations and their relationship to similarity. Students explore the relationship between scale factor and lengths of sides of
figures after dilations. They verify properties of dilations and use dilations to verify similarity. Students prove similarity theorems and use similarity concepts in problem-solving situations (MP1). Students work to prove the midsegment theorem and the side-splitter theorem and its converse. They then apply the theorems to set up and solve proportions to find measures of parts of triangles. Students identify similarity relationships in right triangles and prove the Pythagorean theorem. The unit ends with students exploring directed line segments and how to partition them.

UNIT 6 FOCUS STANDARDS

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

In the following table, green highlights indicate major work of the grade, blue highlights indicate supporting work, and yellow highlights indicate additional work.

<table>
<thead>
<tr>
<th>Standard Text</th>
<th>Standard ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.</td>
<td>GM:G-SRT.A.1.a</td>
</tr>
<tr>
<td>The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</td>
<td>GM:G-SRT.A.1.b</td>
</tr>
<tr>
<td>Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</td>
<td>GM:G-SRT.A.2</td>
</tr>
<tr>
<td>Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</td>
<td>GM:G-SRT.A.3</td>
</tr>
<tr>
<td>Prove and apply theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity; SAS similarity criteria; SSS similarity criteria; AA similarity criteria.</td>
<td>GM:G-SRT.B.4</td>
</tr>
<tr>
<td>Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</td>
<td>GM:G-SRT.B.5</td>
</tr>
<tr>
<td>Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3 ) lies on the circle centered at the origin and containing the point (0, 2).</td>
<td>GM:G-GPE.B.4</td>
</tr>
<tr>
<td>Find the point on a directed line segment between two given points that partitions the segment in a given ratio.</td>
<td>GM:G-GPE.B.6</td>
</tr>
</tbody>
</table>
**Standard Text**

Prove and apply theorems about triangles. *Theorems include:* measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

**Standard ID**

<table>
<thead>
<tr>
<th>GM:G-CO.C.10</th>
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### Unit 6 Pacing Guide

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
</table>
| Dilations | - Verify experimentally the properties of dilations given a center and a scale factor.  
- Calculate and interpret the scale factor for dilations of figures.  
- Determine the unknown measures of an image or pre-image of a dilated figure given the scale factor. | GM:G-SRT.A.1.a, GM:G-SRT.A.1.b | 2              |
| Similar Figures | - Verify the properties of dilations, including the scale factor and slopes of corresponding line segments.  
- Determine if two polygons are similar using dilations.  
- Find the coordinates of the vertices of an image or pre-image of a dilated polygon given the scale factor. | GM:G-SRT.A.1.a, GM:G-SRT.A.1.b, GM:G-SRT.A.2 | 2              |
| Triangle Similarity: AA | - Identify the composition of similarity transformations in a mapping of two triangles.  
- Complete the steps to prove triangles are similar using the AA similarity theorem. | GM:G-SRT.B.4, GM:G-SRT.A.3, GM:G-SRT.A.2, GM:G-SRT.B.5 | 2              |
| Triangle Similarity: SSS and SAS | - Identify the sides and angle that can be used to prove triangle similarity using SSS similarity theorem and SAS similarity theorem.  
- Complete the steps to prove triangles are similar using SAS similarity theorem.  
- Complete the steps to prove triangles are similar using SSS similarity theorem. | GM:G-SRT.B.4, GM:G-SRT.B.5 | 2              |
## Using Triangle Similarity Theorems
- Complete the steps to prove theorems involving similar triangles.
- Solve for unknown measures of similar triangles using the side-splitter theorem and its converse.
- Solve for unknown measures of similar triangles using the triangle midsegment theorem.

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GM:G-SRT.B.4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>GM:G-CO.C.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GM:G-SRT.B.5</td>
<td></td>
</tr>
</tbody>
</table>

## Right Triangle Similarity
- Complete the steps to prove the Pythagorean theorem using similar triangles.
- Identify similar right triangles formed by an altitude and write a similarity statement.
- Apply theorems to solve problems involving geometric means.
- Apply the Pythagorean theorem to find side lengths of a right triangle.

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GM:G-SRT.B.4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>GM:G-SRT.B.5</td>
<td></td>
</tr>
</tbody>
</table>

## Congruent and Similar Triangles in the Coordinate Plane
- Apply coordinate geometry to prove properties of congruent triangles.
- Apply coordinate geometry to prove properties of similar triangles.

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>GM:G-SRT.B.5</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>GM:G-GPE.B.4</td>
<td></td>
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</tbody>
</table>

## Directed Line Segments and Modeling
- Find the coordinates of a point on a directed line segment that partitions the segment into a given ratio.
- Model and solve real-world problems involving directed line segments.

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GM:G-GPE.B.6</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>GM:G-MG.A.3</td>
<td></td>
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</tbody>
</table>

## Unit Test

<table>
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<tr>
<th>Number of Days</th>
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<tbody>
<tr>
<td>1</td>
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</tbody>
</table>

### Discussion Questions & Answers

1. One student says that the angles of similar figures are proportional while another say that the angles are not proportional but congruent. Apply what you know about similar and congruent to help clarify the true relationship among the angles of similar figures.

   a. **The angles of similar figures are proportional with a ratio of 1. In fact, it can be said that all congruent shapes are also similar. However, not all similar shapes are congruent since not all similar shapes have a scale factor of 1.**

2. Describe a dilation where the image and pre-image share a vertex. Generalize the criteria for when this type of dilation occurs.

   a. **The dilation would include a pre-image and image with a shared vertex. A simple example would be a triangle with its midsegment drawn. Whether the pre-image is the larger undergoing a reduction or the pre-image is the smaller undergoing an**
enlargement, the two figures can share one vertex. The generalization should include a reference to the center of the dilation being at the shared vertex.

3. Draw a triangle and its three midsegments. Analyze the triangles to describe the relationship among all five triangles in the figure.
   a. The triangle whose sides are the midsegments is similar to the original triangle because each midsegment is half the length of the side it does not intersect. The triangles created with a midsegment as their base that also share an angle with the original triangle are also similar to the original triangle using corresponding angles with the midsegment and the side of the original triangle it is parallel to. The four smaller triangles are also congruent to one another using the SSS theorem because each has one side that is half the length of each of the original sides.

Common Misconceptions

- Dilations
  - Students may believe that to have a reduction, the scale factor must be negative rather than correctly understanding that the scale factor must be between 0 and 1 to create the reduction.

- Similarity statements
  - Students may forget the importance of the order of vertices when making similarity statements. The corresponding vertices need to be in the same sequence for the statement to be true.

- Proving congruency
  - Students may not recognize when they are given congruent corresponding parts that match specific triangle congruence theorems.

Classroom Challenge

For each of the proofs, determine what information was given, including both a diagram and the necessary mathematical statement(s).

1. 

<table>
<thead>
<tr>
<th>Proof</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Statement</td>
<td>Reason</td>
</tr>
<tr>
<td>1.</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( \angle A \cong \angle A )</td>
<td>Reflexive property</td>
</tr>
<tr>
<td>3. ( \triangle AED \cong \triangle ACB ) and ( \angle ADE \cong \angle ABC )</td>
<td>Corresponding Angles Postulate</td>
</tr>
<tr>
<td>4. ( \triangle AED \sim \triangle ACB )</td>
<td>AA Similarity Postulate</td>
</tr>
</tbody>
</table>

2. 

<table>
<thead>
<tr>
<th>Proof</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Statement</td>
<td>Reason</td>
</tr>
<tr>
<td>1.</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( \angle BEA \cong \angle CED )</td>
<td>Vertical Angles Theorem</td>
</tr>
</tbody>
</table>
Possible solution pathway:

1. To use the reflexive property, the triangles must be superimposed. To use the corresponding angles postulate, the diagram must include a pair of parallel lines. Combine the diagram with the statement $\overline{DE} \parallel \overline{BC}$ for the given information of the proof.

   ![Diagram 1](image1)

2. To use the vertical angle theorem, the diagram must include vertical angles. To use the SAS theorem, we need to have two pairs of proportional sides. Combine the diagram with the statement $\frac{\overline{BE}}{\overline{AE}} = \frac{\overline{ED}}{\overline{EC}}$ for the given information of the proof.

   ![Diagram 2](image2)

Teacher notes:

For struggling students, ask them to identify the scenarios where each reason appears. Sketches of these situations can help with the visualization of the criteria within the triangles.

For advanced students, ask them to describe a group of transformations that would lead to the diagrams for each of the proofs.

**UNIT 7: RIGHT TRIANGLE RELATIONSHIPS AND TRIGONOMETRY**

*Estimated Unit Time: approx. 11 Class Periods total*

In the unit Right Triangle Relationships and Trigonometry, students work to discover multiple methods for solving right triangles. They begin by classifying triangles and applying the converse of the Pythagorean theorem and triangle inequality theorems to solve problems. Students build an
understanding of the meanings of trigonometric ratios based on their knowledge of similar triangles. They identify trigonometric ratios in problem-solving situations (MP1). They model real-world problems using equations of trigonometric ratios and solve these problems to find missing side lengths and angle measures (MP4).

**Unit 7 Focus Standards**

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

In the following table, green highlights indicate major work of the grade, blue highlights indicate supporting work, and yellow highlights indicate additional work.

<table>
<thead>
<tr>
<th>Standard Text</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</td>
<td>GM:G-MG.A.1</td>
</tr>
<tr>
<td>Understand that by similarity, side ratios in right triangles, including special right triangles (30-60-90 and 45-45-90), are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</td>
<td>GM:G-SRT.C.6</td>
</tr>
<tr>
<td>Explain and use the relationship between the sine and cosine of complementary angles.</td>
<td>GM:G-SRT.C.7</td>
</tr>
<tr>
<td>Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</td>
<td>GM:G-SRT.C.8</td>
</tr>
</tbody>
</table>

**Unit 7 Pacing Guide**

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
</table>
| Triangle Classification Theorems | - Classify a triangle using the converse of the Pythagorean theorem and triangle inequality theorems.  
- Apply the converse of the Pythagorean theorem and triangle inequality theorems to solve problems.  
- Determine an unknown side length or range of side lengths of a triangle given its classification. | GM:G-MG.A.1   | 2              |
### Lesson Objectives

#### Special Right Triangles
- Complete the steps to prove special right triangle theorems.
- Determine unknown measures of 45°-45°-90° triangles.
- Determine unknown measures of 30°-60°-90° triangles.
- Solve real-world problems involving special right triangles.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Number of Days</th>
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</thead>
<tbody>
<tr>
<td>GM:G-MG.A.1</td>
<td>2</td>
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</tbody>
</table>

#### Trigonometric Ratios
- Given an acute angle of a right triangle, label the hypotenuse, opposite, and adjacent sides.
- Given an acute angle of a right triangle, write ratios for sine, cosine, and tangent.
- Relate trigonometric ratios of similar triangles and the acute angles of a right triangle.

<table>
<thead>
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<tbody>
<tr>
<td>GM:G-SRT.C.6</td>
<td>2</td>
</tr>
<tr>
<td>GM:G-SRT.C.7</td>
<td></td>
</tr>
</tbody>
</table>

#### Solving for Side Lengths of Right Triangles
- Write equations using trigonometric ratios that can be used to solve for unknown side lengths of right triangles.
- Solve for unknown side lengths of right triangles using trigonometric ratios.
- Apply trigonometric ratios to solve real-world problems.

<table>
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<tr>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM:G-SRT.C.8</td>
<td>2</td>
</tr>
</tbody>
</table>

#### Solving for Angle Measures of Right Triangles
- Write equations that can be used to solve for unknown angles in right triangles.
- Solve for unknown angles of right triangles using inverse trigonometric functions.
- Apply inverse trigonometric functions to solve real-world problems.

<table>
<thead>
<tr>
<th>Standards</th>
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</thead>
<tbody>
<tr>
<td>GM:G-SRT.C.8</td>
<td>2</td>
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</tbody>
</table>

### Discussion Questions & Answers

1. Without completing the entire proof, summarize how the relationship of the sides and angles of a right triangle logically leads to the acute triangle inequality theorem and the obtuse triangle inequality theorem.
   
   a. **Start with the acute triangle inequality theorem.** In a right triangle, the hypotenuse is opposite the longest side. This means that if the square of the longest side is less than the square of the hypotenuse when it is a right triangle, it makes sense that the angle is less than 90° based on understanding that longer side lengths are opposite larger angles. The same logic holds true for the obtuse triangle inequality theorem.
2. Hypothesize about why either the sine or cosine be used to find the length of one leg of a right triangle given the hypotenuse and one acute angle.
   a. One of the trigonometric functions will be obvious based on the given acute angle since the leg will be either the opposite or the adjacent side to that angle. Since the acute angles in a right triangle are complementary, the unused trigonometric function could be used after finding the third angle in the right triangle.

3. The lesson uses the alternate interior angles theorem to show the relationship between the angle of depression and the angle of elevation. Examine these angles to determine another way to show the angles are congruent.
   a. Draw a perpendicular transversal that intersects the bottom horizontal line at the vertex of the angle of elevation. These same side interior angles create a right angle that includes the angle of elevation. The angle of depression is now one acute angle in a right triangle where the acute angles are always complementary. The angle of elevation is complementary to the same angle that the angle of depression is complementary to so the angle of elevation and the angle of depression are equivalent.

Common Misconceptions

- Special right triangles
  o Students may confuse the ratios of the side lengths for special right triangles.
  o Students may try to apply these ratios to right triangles that do not meet the criteria for special right triangles.

- Trigonometric ratios
  o Students may not realize that the labels of legs as opposite or adjacent change depending on the referenced acute angle.
  o Students may try to use trigonometric ratios for acute or obtuse triangles, not realizing they are limited to use with right triangles.

- Naming angles
  o Students may confuse the definition of the angle of elevation and the angle of depression when solving real-world applications of trigonometric functions.
Classroom Challenges

Figure ABCD is a rectangle with sides DC = 24 and BC = 18.
1. Find the lengths of sides AC, HJ, JG, and JC.
2. Find \( m\angle AJD, m\angle CJG, m\angle FJC, \) and \( m\angle JGC. \)

Possible solution pathway:

1. \( \overline{AC} \) is a diagonal of the rectangle and the hypotenuse of right triangle ABC (or ABD) with base lengths of 24 and 18.
   \[
   24^2 + 18^2 = AC^2 \\
   576 + 324 = AC^2 \\
   900 = AC^2 \\
   30 = AC
   \]
   \( \overline{HJ} \) is parallel to \( \overline{DC} \) and goes from side AD to the intersection of the diagonals, which is at the center of the sides of the rectangle.
   \( HJ = 0.5(24) = 12 \)

2. \( \overline{JG} \) is parallel to \( \overline{BC} \) and goes from side DC to the intersection of the diagonals, which is at the center of the sides of the rectangle.
   \( JG = 0.5(18) = 9 \)

3. \( \overline{JC} \) is the hypotenuse of right triangle JGC with base lengths 12 and 9.
   \[
   12^2 + 9^2 = JC^2 \\
   144 + 81 = JC^2 \\
   225 = JC^2 \\
   15 = JC
   \]

2. \( \angle AJD \) is comprised of two angles that are congruent to \( \angle FJC. \) To find \( m\angle FJC, \) let \( m\angle FJC = x. \)
\[
\tan x = \frac{9}{12} \\
\tan x = 0.75 \\
x = 64^\circ = m \angle FJC
\]

Using the relationship already stated: \( m \angle AJD = 2m \angle FJC = 2(64^\circ) = 128^\circ \)

By alternate interior angles, \( m \angle JGC = 90^\circ \).

According to the diagram, \( \angle FJC \) and \( \angle CJF \) are complementary.

\[
90^\circ - m \angle FJC = m \angle CJF \\
90^\circ - 64^\circ = 26^\circ = m \angle CJF
\]

**Teacher notes:**

Ask struggling students to identify congruent triangles that contain the angles whose measure needs to be determined. Then have students draw these triangles separate from the given diagram and label all the parts in both the rectangle and the individual triangles.

Advanced students can double and half the given lengths of the rectangle, to write a generalization about how the angles change in each case.

**UNIT 8: QUADRILATERALS AND COORDINATE ALGEBRA**

*Estimated Unit Time: approx. 13 Class Periods total*

In the unit Quadrilaterals and Coordinate Algebra, students classify and describe four-sided figures. They build on their knowledge of classifying figures from previous grades. Students complete the steps to prove theorems about properties of parallelograms. They then use these properties to solve real-world and mathematical problems involving angle measures and measures of segments along sides and diagonals of parallelograms. Students apply coordinate algebra proofs to triangles and quadrilaterals. They use their knowledge of the distance and slope formulas from Algebra I to solve the problems.

**Unit 8 Focus Standards**

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

In the following table, **green highlights** indicate major work of the grade, **blue highlights** indicate supporting work, and **yellow highlights** indicate additional work.

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<tr>
<td>Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</td>
<td>GM:G-MG.A.1</td>
</tr>
</tbody>
</table>
## Standard Text

Prove and apply theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point \((1, \sqrt{3})\) lies on the circle centered at the origin and containing the point \((0, 2)\).

Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

## Unit 8 Pacing Guide

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
</table>
| **Classifying Quadrilaterals** | • Classify and describe relationships within the family of quadrilaterals.  
                                | • Describe real-world objects using characteristics of quadrilaterals.  
                                | • Solve mathematical problems using characteristics of quadrilaterals.  
                                | • Solve real-world problems using characteristics of quadrilaterals.  
                                | GM:G-CO.C.11, GM:G-MG.A.1 | 2                 |
| **Parallelograms**            | • Complete the steps to prove theorems about properties of parallelograms.  
                                | • Apply properties of parallelograms to solve problems.  
                                | GM:G-CO.C.11         | 2               |
| **Proving a Quadrilateral Is a Parallelogram** | • Complete the steps to prove that a quadrilateral is a parallelogram.  
                                | • Apply properties of parallelograms to solve for unknown values.  
                                | • Analyze a figure to determine if it is a parallelogram.  
                                | GM:G-CO.C.11         | 2               |
| **Special Parallelograms**    | • Complete the steps to prove theorems about properties of parallelograms.  
                                | • Apply properties of rhombi to solve mathematical and real-world problems.  
                                | • Apply properties of rectangles to solve mathematical and real-world problems.  
                                | • Apply properties of squares to solve mathematical and real-world problems.  
                                | GM:G-CO.C.11, GM:G-MG.A.1 | 2               |
**Lesson Objectives**

- **Trapezoids and Kites**
  - Complete proofs involving properties of trapezoids and kites.
  - Apply properties of trapezoids to solve mathematical and real-world problems.
  - Apply properties of kites to solve mathematical and real-world problems.

- **Figures in the Coordinate Plane**
  - Apply coordinate algebra proofs to triangles and quadrilaterals.
  - Calculate the perimeter of a triangle or quadrilateral given the coordinates of the vertices.

**Standards**

- GM:G-CO.C.11
- GM:G-MG.A.1
- GM:G-P.E.B.4
- GM:G-P.E.B.7

**Number of Days**

- 2
- 2

---

**Discussion Questions & Answers**

1. How are congruence and equal measure related? How can figures on a coordinate plane be shown to be congruent using the distance formula?
   
   a. *Congruence is a physical idea that states that you can place two congruent figures on top of each other and the edges and vertices match up exactly. In this way, a figure and its reflection are congruent because you can fold one figure over the other and have an exact match. Congruent figures also have equal measures for corresponding edges and angles. It is possible to develop geometric notions and proofs referring only to congruence and not actually measuring figures.*

   b. *Since congruent figures have equal measures for corresponding edges and angles, any two figures placed on a coordinate plane can have their edge lengths measured using the distance formula. If the corresponding edges of two figures placed on a coordinate plane have the same measure, then the two figures are congruent.*

2. Compare and contrast a kite with an isosceles trapezoid.

   a. *A kite has two pairs of adjacent congruent sides. The pairs of congruent sides have different lengths from each other. An isosceles trapezoid has two bases that are different lengths and two congruent sides that are not bases. The diagonals of a kite are not congruent, but they are perpendicular to each other. The diagonals of an isosceles trapezoid are congruent, but they are not perpendicular to each other.*

3. A figure can be classified as a quadrilateral, a parallelogram, and a rectangle, but it cannot be classified as a rhombus. Explain why “rectangle” is the best and most precise name for the figure.

   a. *If the figure has equal side lengths, then it could be classified as a rhombus and as a square. Since it cannot be classified as a rhombus, then it does not have equal side...*
lengths. Parallelogram is too general a classification because it does not take into consideration that the figure also has 4 right angles. There are parallelograms that do not have 4 right angles. So, rectangle is the best and most precise name for the figure.

Common Misconceptions

- Congruence
  - Students may not grasp the subtle difference between congruence and equal measure.
  - Students may assume two segments or angles that look the same are congruent, even if they are not marked.
  - Students may incorrectly match up corresponding parts when determining the congruence of two figures.

- Classification of quadrilaterals
  - Students may not understand that certain quadrilaterals can be classified in different ways and can belong to different overlapping subcategories.
  - Students may incorrectly apply the attributes of a subcategory of quadrilaterals to all quadrilaterals.

- Proof
  - Students may not understand the idea of necessary and sufficient requirements to prove that a quadrilateral belongs in a particular category.
  - Students may think that one or two specific examples of a conjecture are sufficient to prove that the conjecture can be generalized.

- Notation
  - Students may become confused by the marks on figures indicating congruent sides and angles, parallel lines, and right angles.
Classroom Challenges

Complete the flowchart with the most specific name of the polygon after each change and a change that results in a quadrilateral with the given name.

Possible solution pathway:

Start with a parallelogram.

Restrict one angle to 90°.
- Name:

Change the shape so that it has a pair of adjacent sides that are congruent.
- Name: Rectangle

Change the shape so that it has a pair of adjacent sides that are congruent.
- Name: Square

Change: Double the lengths of two adjacent sides maintaining perpendicular diagonals.
- Kite
Teacher notes:

For struggling visual learners ask them to sketch shapes with the criteria. Students can also list special quadrilaterals and their characteristics.

For advanced students, ask them to create a second flowchart that starts with a square and ends with a trapezoid with at least two changes. It also might be interesting to start the same flowchart that is given with a quadrilateral instead of a parallelogram.

UNIT 9: CIRCLES

Estimated Unit Time: approx. 19 Class Periods

In the unit Circles, students build a solid understanding of parts of circles and the different types of angles that can be formed by lines and line segments related to circles. Students prove that all circles are similar. They calculate measures of central angles, chords, and arcs and solve problems using the relationship between the radius and tangent of a circle. Students prove circle congruency theorems and theorems involving inscribed angles and their intercepted arcs, and they use these theorems to solve problems (MP1). Students solve problems relating to radius, circumference, arc length, and areas of sectors. Students convert between degree and radian measures of angles and use radians to solve circle measurement problems. Students perform and justify constructions of a circle from three points that are not collinear, construct regular polygons inscribed in circles, and construct tangent lines. Students use the Pythagorean theorem and the definition of a circle to derive the equation of a circle. The unit ends with a performance task that relates major concepts about triangles to the circle concepts learned in this unit.

Unit 9 Focus Standards

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

In the following table, green highlights indicate major work of the grade, blue highlights indicate supporting work, and yellow highlights indicate additional work.

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<td>------------------</td>
</tr>
<tr>
<td>Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point ((1, \sqrt{3})) lies on the circle centered at the origin and containing the point ((0, 2)).</td>
<td>GM:G-GPE.B.4</td>
</tr>
<tr>
<td>Prove and apply theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity; SAS similarity criteria; SSS similarity criteria; AA similarity criteria.</td>
<td>GM:G-SRT.B.4</td>
</tr>
<tr>
<td>Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</td>
<td>GM:G-SRT.C.8</td>
</tr>
<tr>
<td>Prove and apply theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.</td>
<td>GM:G-CO.C.9</td>
</tr>
<tr>
<td>Prove and apply theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</td>
<td>GM:G-CO.C.11</td>
</tr>
<tr>
<td>Prove that all circles are similar.</td>
<td>GM:G-C.A.1</td>
</tr>
<tr>
<td>Identify and describe relationships among inscribed angles, radii, and chords, including the following: the relationship that exists between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; and a radius of a circle is perpendicular to the tangent where the radius intersects the circle.</td>
<td>GM:G-C.A.2</td>
</tr>
<tr>
<td>Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.</td>
<td>GM:G-C.A.3</td>
</tr>
<tr>
<td>Give an informal argument, e.g., dissection arguments, Cavalieri’s principle, or informal limit arguments, for the formulas for the circumference of a circle; area of a circle; volume of a cylinder, pyramid, and cone.</td>
<td>GM:G-GMD.A.1</td>
</tr>
<tr>
<td>Use similarity to determine that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</td>
<td>GM:G-C.B.5</td>
</tr>
<tr>
<td>Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</td>
<td>GM:G-GPE.A.1</td>
</tr>
<tr>
<td>Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.</td>
<td>GM:G-CO.D.13</td>
</tr>
<tr>
<td>Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</td>
<td>GM:G-CO.A.1</td>
</tr>
</tbody>
</table>
## Unit 9 Pacing Guide

<table>
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<tr>
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| **Introduction to Circles** | • Complete the steps to prove that all circles are similar.  
• Identify and describe terms related to circles.  
• Calculate the degree measure of an arc using the arc addition postulate. | GM:G-C.A.1                  | 1              |
| **Central Angles**      | • Identify congruent central angles, chords, and arcs.  
• Determine the measures of central angles, chords, and arcs using theorems about angle, chord, and arc congruency.  
• Solve problems using the radius tangent theorem and its converse. | GM:G-C.A.2                  | 1              |
| **Inscribed Angles**    | • Complete the steps to prove theorems involving inscribed angles and their intercepted arcs.  
• Apply theorems about inscribed angles and angles formed by a tangent and a chord. | GM:G-C.A.2  
GM:G-C.A.3 | 1              |
| **Secants, Tangents, and Angles** | • Solve problems involving angles formed by two intersecting chords.  
• Solve problems involving angles formed by two secants that intersect outside a circle.  
• Solve problems involving angles formed by two intersecting tangents.  
• Solve problems involving angles formed by a secant and a tangent that intersect outside a circle. | GM:G-C.A.2                  | 1              |
| **Special Segments**    | • Solve problems involving segments formed by two intersecting chords.  
• Solve problems involving segments formed by two secants, which intersect outside a circle.  
• Solve problems involving segments formed by two intersecting tangents.  
• Solve problems involving segments formed by a secant and a tangent, which intersect outside a circle. | GM:G-MG.A.1  
GM:G-C.A.2 | 1.5            |
| **Circumference and Arc Length** | • Solve problems involving circumference of a circle.  
• Determine the radian measure of a central angle.  
• Solve problems involving arc length with central angles measured in degrees.  
• Solve problems involving arc length with central angles measured in radians. | GM:G-MG.A.1  
GM:G-C.A.2  
GM:G-GMD.A.1  
GM:G-C.B.5 | 2              |
### Lesson Objectives

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| Area of a Circle and a Sector  | • Solve problems involving area of a circle.  
• Solve problems involving area of a sector with central angles measured in radians.  
• Solve problems involving area of a sector with central angles measured in degrees.                                                   | GM:G-GMD.A.1  
GM:G-G.C.B.5         | 2 |
| Angle Relationships            | • Determine segment lengths, angle measures, and arc measures using definitions and theorems relating to circles.                                                                                  | GM:G-C.A.2               | 1.5           |
| Performance Task: Circle Constructions |                                                                                                                                                                                                                 | GM:G-CO.D.13             | 2             |
| Equation of a Circle           | • Identify the center and radius from the equation of a circle, including equations given in general form.  
• Determine the equation of a circle.  
• Determine if a given point lies on a circle.                                                                 | GM:G-GPE.B.4  
GM:G-GPE.A.1         | 2             |
| Performance Task: Common Tangents of Circles |                                                                                                                                                                                                                 | GM:G-SRT.B.4  
GM:G-CO.C.11  
GM:G-CO.C.9  
GM:G-SRT.C.8  
GM:G-CO.A.1  
GM:G-C.A.2       | 3             |
| Unit Test                      |                                                                                                                                                                                                              |                          | 1             |

### Discussion Questions & Answers

1. Describe the relationships among the terms chord, diameter, and radius.
   
   a. A chord is the segment formed between two points on the circle. A chord is part of a secant. Chords can pass through any part of the circle. A chord that passes through the center point of a circle can be further described as a diameter. Since an infinite number of chords can be drawn from two points on the circle through the center of a circle, a circle has an infinite number of possible chords that are diameters. However, the length of the diameter is always the same number. A radius is a line segment that connects the center of the circle with a point on the circle. The center of a circle divides a diameter into two radii. The length of a radius is always half the length of the diameter.

2. Explain how to use the standard form of an equation of a circle to find the center point and length of the radius. If you are given the length of the radius and the coordinates for the center of the circle, explain how to write an equation for the circle in standard form. Give an example.
a. The standard form for the equation of a circle is \((x - h)^2 + (y - k)^2 = r^2\). When the equation is placed in this form, the point \((h, k)\) is the center of the circle. The square root of \(r^2\) will give the length of the radius, \(r\).

b. If you are given the point representing the center of the circle and the length of the radius, you have the numbers corresponding to \(h\), \(k\), and \(r\) in the standard form of the equation. You can substitute these numbers directly into the form, being careful about the signs of the numbers \(h\) and \(k\). For example, suppose you have a circle with a center at \((-4, 3)\) and a radius of 7. Since \(h = -4\), \(k = 3\), and \(r = 7\), the equation could be written as \((x + 4)^2 + (y - 3)^2 = 49\).

3. If you double the radius of a circle, what is the effect on the area of the circle?

   a. The formula for the area of a circle is \(A = \pi r^2\). Since the radius is squared in the formula, if you multiply \(r\) by 2, the area will increase by \(2^2 = 4\) times.

4. Explain how you can use the circumcenter of a triangle to help you draw a circle that inscribes the triangle in the circle.

   a. The circumcenter of a triangle is a point that is equidistant from each of the vertices of the triangle. If you place a compass point on the circumcenter and the drawing point on one of the vertices, you can draw a circle that will include all three vertices of the triangle. This is because the opening of the compass is set to the radius of a circle that consists of all points equidistant from the circumcenter of the triangle. This circle, therefore, also includes all three of the triangle’s vertices.

Common Misconceptions

- Similarity and congruence.
  - Students may not understand that all circles are similar.
  - Students may incorrectly think that because all circles are similar, then all circles are congruent.

- \(\pi\)
  - Students may not understand that \(\pi\) is an irrational transcendental number and that all numbers used in place of \(\pi\) in calculations are approximations (including the truncated \(\pi\) used on their calculators).
  - Students may not understand that a value for circumference or area of a circle left in terms of \(\pi\) is more accurate than one that uses approximation.
  - Students may incorrectly think of \(\pi\) as a variable instead of as a symbol for a specific number.
• Attributes of circles
  o Students may confuse chords, secants, and tangents.
  o Students may mix up inscribed and circumscribed.
  o Students may confuse arc length, central angle measurement, and degree measure of an arc.

• Tangent and secant
  o Students may not understand that a tangent has a point of tangency at exactly and only one point.
  o Students may not remember that a secant intersects a circle at exactly two points.

Classroom Challenge

Start with circle A, where $\overline{AB}$ is the radius of A, and circle F, where $\overline{DC}$ is the diameter of F. Also, consider parallelogram ABCD.

1. Sketch circles A and F and parallelogram ABCD. Describe the relative sizes of circle A and circle F using the properties of the parallelogram ABCD.

2. If $\overline{BC}$ is tangent to circle A at point B, write at least two statements about the characteristics of the parallelogram.

3. Use the parallelogram created in 2. Find the ratio that compares the area of the sector of circle A that intersects the parallelogram to the area of the sector of circle F that intersects the rectangle.

Possible solution pathway:

Since the opposite sides of the parallelogram are congruent, the length of radius of circle A is equal to the length of the diameter of circle F. This means the radius of circle A is twice the radius of circle F. This means that the area of circle A is four times the area of circle F.
2. If $\overline{BC}$ is tangent to circle $A$, line segments $\overline{AB}$ and $\overline{BC}$ are perpendicular. Since $\overline{AB}$ is parallel to $\overline{CD}$, then $\overline{CD}$ is perpendicular to $\overline{BC}$. If $\overline{FC}$ is perpendicular to $\overline{AB}$, then $\overline{AB}$ is tangent to circle $F$ at point $C$.

3. Let $r$ be the radius of circle $F$. Based on the work in question 1, the radius of circle $A$ is $2r$. The area of circle $F$ is $\pi r^2$. The area of circle $A$ is $\pi (2r)^2$ or $4\pi r^2$. Half of circle $F$ intersects the rectangle created in 2. This sector has an area of $\frac{1}{2}\pi r^2$. One-quarter of circle $A$ intersects the rectangle created in 2. This sector has an area of $\frac{1}{4} (4\pi r^2)$ or $\pi r^2$. The ratio of the area of the sector of circle $A$ that intersects the rectangle to the area of the sector of circle $F$ that intersects the rectangle is 2:1.

**Teacher notes:**

For struggling students, ask the students to write the properties of all parallelograms. Also, help students define radius and diameter. Encourage visual learners to sketch the parallelogram and then add the circles to the sketch or to use a coordinate grid like the one in the possible solutions pathway.

For advanced students, ask what happens if the parallelogram is a square.

**UNIT 10: GEOMETRIC MODELING IN TWO DIMENSIONS**

*Estimated Unit Time: approx. 16 Class Periods*

In the unit Geometric Modeling in Two Dimensions, students develop methods for finding areas of polygons using formulas and relationships found when polygons are graphed in the coordinate plane. They use their knowledge of finding areas of figures from previous grade levels and the distance formula from Algebra I. Students apply the distance formula to calculate perimeters of trapezoids, rhombi, parallelograms, and other polygons in the coordinate plane. They apply interior and exterior angle sum theorems for polygons to solve problems. Students use critical thinking skills as they apply previous work with the Pythagorean theorem to find apothem lengths when determining areas of regular polygons. The unit ends with a lesson on modeling density and design problems using geometric concepts (MP4).

**Unit 10 Focus Standards**

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.
In the following table, **green highlights** indicate major work of the grade, **blue highlights** indicate supporting work, and **yellow highlights** indicate additional work.

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<td>GM:G-MG.A.1</td>
</tr>
<tr>
<td>Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).</td>
<td>GM:G-MG.A.2</td>
</tr>
<tr>
<td>Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).</td>
<td>GM:G-MG.A.3</td>
</tr>
<tr>
<td>Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.</td>
<td>GM:G-GPE.B.7</td>
</tr>
<tr>
<td>Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</td>
<td>GM:G-SRT.C.8</td>
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### Unit 10 Pacing Guide

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<td><strong>Area of Triangles and Parallelograms</strong></td>
<td>Solve problems involving areas of triangles and parallelograms.</td>
<td>GM:G-GPE.B.7</td>
<td>2</td>
</tr>
<tr>
<td><strong>Perimeter and Area of Rhombi, Trapezoids, and Kites</strong></td>
<td>● Solve problems involving the area of a rhombus, trapezoid, and kite.</td>
<td>GM:G-GPE.B.7, GM:G-MG.A.1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>● Solve problems involving the area of a rhombus, trapezoid, and kite given the coordinates of the vertices.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>● Calculate the perimeter of a rhombus, trapezoid, or kite given the coordinates of the vertices.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Angle Measures of Polygons</strong></td>
<td>● Identify and describe polygons.</td>
<td>GM:G-MG.A.1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>● Apply the polygon interior angle sum theorem to solve problems.</td>
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<tr>
<td></td>
<td>● Apply the polygon exterior angle sum theorem to solve problems.</td>
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<tr>
<td><strong>Area of Regular Polygons</strong></td>
<td>● Calculate the length of the apothem of a regular polygon.</td>
<td>GM:G-MG.A.1, GM:G-SRT.C.8</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>● Calculate the area of a regular polygon.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>● Solve real-world problems involving the area of regular polygons.</td>
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</tbody>
</table>
## Lesson Objectives

### Area of Composite Figures
- Decompose composite two-dimensional figures.
- Calculate the area of composite two-dimensional figures, including real-world applications.
- Write an expression that represents the area of a composite two-dimensional figure.

<table>
<thead>
<tr>
<th>Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM:G-MG.A.1</td>
</tr>
</tbody>
</table>

### Density and Design Problems
- Solve problems involving density of an area.
- Use geometric concepts to solve design problems.

<table>
<thead>
<tr>
<th>Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM:G-MG.A.2</td>
</tr>
<tr>
<td>GM:G-MG.A.3</td>
</tr>
</tbody>
</table>

### Performance Task: Tessellations

<table>
<thead>
<tr>
<th>Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM:G-MG.A.3</td>
</tr>
</tbody>
</table>

### Unit Test

<table>
<thead>
<tr>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

## Discussion Questions & Answers

1. Compare and contrast finding the length of a segment from point (3, 0) to point (7, 0) and finding the length of a segment from point (3, 0) to point (0, 4). Then describe how to find the perimeter and area of a parallelogram with vertices at (3, 0), (7, 0), (4, 4), and (0, 4).

   a. *Both lengths will be found in terms of the units represented on the coordinate plane. Because the points (3, 0) and (7, 0) lie on the same vertical line, the distance can be found by subtracting 7 – 3 = 4. The length of the segment is 4 units. Points (3, 0) and (0, 4) do not lie on a vertical or horizontal line. The distance formula will be needed to find the distance between the points (Distance = \(\sqrt{(0 - 3)^2 + (4 - 0)^2} = \sqrt{9 + 16} = \sqrt{25} = 5\)).*

   b. *To find the perimeter, determine the lengths of the sides. From part a of the question, we know that the segment from (3, 0) to (7, 0) has a length of 4, and the segment from (3, 0) to (0, 4) has a length of 5. Similarly, because parallelograms have parallel and congruent opposite sides, the segment from (0, 4) to (4, 4) has a length of 4. And the segment from (4, 4) to (7, 0) has a length of 5. This can be confirmed with the distance formula. Adding the lengths gives a perimeter of 18 units. Letting the segment from (3, 0) to (7, 0) be the base, the height of the parallelogram is 4 units. Using the formula base \(\times\) height = area, we multiply 4 \(\times\) 4 = 16, for an area of 16 square units.*

2. Describe the difference between the radius and the apothem of a regular polygon. How can the apothem be used to find the area of a regular polygon?

   a. *The radius of a regular polygon is the distance from the center of the polygon to any vertex. The apothem of a regular polygon is the distance from the center of a regular polygon to the midpoint of any of the polygon’s sides.*

   b. *For a regular polygon, use the formula \(A = \frac{1}{2}ap\), where \(A\) is area, \(a\) is the length of the apothem, and \(p\) is the perimeter of the polygon.*
3. One 200 square-mile section of a national park has an estimated elk population of 2,100. Another 450 square-mile section of the park has an estimated elk population of 5,900. In which section of the park is a visitor more likely to see an elk? Explain why.

   a. The density of elk in a given area is found by dividing the total number of elk by the area. For the 200 square-mile section, the density is $2100 \div 200 = 10.5$ elk per square mile. For the 450 square-mile section, the density is $5900 \div 450 \approx 13.1$ elk per square mile. Since there is a greater density of elk in the 450 square-mile section, a visitor would be more likely to see an elk in this section.

Common Misconceptions

- Area and perimeter
  - Students may incorrectly think that area is fixed no matter what the perimeter.
  - Students may incorrectly think that perimeter is fixed no matter what the area.

- Formulas
  - Students may have difficulty generalizing formulas such as generalizing from area = length $\times$ width for a rectangle to area = base $\times$ height for any parallelogram.
  - Students may not understand that any side of a parallelogram or triangle can be chosen as the base when applying an area formula. The area will be the same no matter which base is chosen.

- Triangle base and height
  - Students may not understand the difference between base and height in a right triangle and base and height in any other kind of triangle.
  - Students may have difficulty identifying the height of a triangle, especially in scalene triangles where the shortest base is oriented horizontally.

- Orientation of the shape
  - Students may think the same figure is a different type of shape when placed in a different orientation. For example, students may incorrectly think that a square that is turned to look like a diamond shape is no longer a square but is now a rhombus.
  - Students may not recognize the right angle in a right triangle when the sides that make up the right angle are not oriented horizontally and vertically.

- Concave and convex
  - Students may confuse concave and convex when describing polygons.

Classroom Challenge

1. Triangle ABC and kite ABDE overlap in such a way that $\overline{AC}$ and $\overline{BC}$ are both contained on diagonals of ABDE. If A is located at (2, 3) and B is located at (5, -1), then what is a possible location for C?
1. Since $\overline{AC}$ and $\overline{BC}$ are both contained on diagonals of a kite and are sides of the triangle ABC, they must meet at a right angle. The two possible locations for point C are (5, 3) and (2, -1).
2. No matter which coordinates are used for C, the base and height of ABC are 3 and 4. This makes the area of ABC equal to 6 square units. Given that triangle ABC is one-third the area of the kite ABDE, we have that $6 = \frac{1}{3} \text{(area ABDE)}$, which means that area ABDE = 18 square units.
3. There are a few possible answers here given that C has two possible locations and it is not given that angles A and D or angles B and E are the non-vertex angles of ABDE. Here is one possible explanation for the locations of D and E given that C is located at (2, -1) and angles B and E are non-vertex angles:

Assume angles B and E are the non-vertex angles for ABDE. This means that the distance from C to E equals the distance from B to C. The length of the diagonal is 6, so E is located at (-1, -1).

The total area of ABDE is 18 square units. Triangle ABC has an area of 6 square units and triangle ACE does as well. That means the other two right triangles created by splitting the kite by the diagonals have a total area of $18 - 12 = 6$ square units total or 3 square units each. Substitute what is known about triangle ECD into the area formula for a triangle to determine CD, which is the height of triangle ECD

\[
\text{Area}_{ECD} = \frac{1}{2}bh
\]

\[
3 = \frac{1}{2}(3)h
\]

\[
2 = h
\]

If CD is 2 units, D must be located at (2, -3).

<table>
<thead>
<tr>
<th>Location of C</th>
<th>Locations of D and E</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5, 3)</td>
<td>If B and E are non-vertex angles: D(6.5, 3) and E(5, 7)</td>
</tr>
<tr>
<td></td>
<td>If A and D are non-vertex angles: D(8, 3) and E(5, 5)</td>
</tr>
<tr>
<td>(2, -1)</td>
<td>If B and E are non-vertex angles: D(2, -3) and E(-1, -1)</td>
</tr>
<tr>
<td></td>
<td>If A and D are non-vertex angles: D(2, -5) and E(0.5, -1)</td>
</tr>
</tbody>
</table>

**Teacher notes:**

Have struggling students write all the properties of a kite and then sketch a kite finding the pairs of right triangles. Visual learners could benefit from shading one of the triangles and designating it.
ABC. From there, move the students to graph paper to use the given information and the generic sketch to develop a kite with the given properties.

For advanced students, have them verify the points they find actually define a shape with two pairs of congruent adjacent sides using the distance formula. Then have them draw a right isosceles triangle and develop the formula.

**UNIT 11: THREE-DIMENSIONAL GEOMETRY**

*Estimated Unit Time: approx. 12 Class Periods*

In the unit Three-Dimensional Geometry, students explore relationships between two- and three-dimensional figures. They solve for unknown side lengths of right triangles within a cube. Students explore the polygons formed when planes intersect solids and the solids created when polygons are rotated about lines that intersect the polygons. Students classify solids and identify parts such as bases, faces, edges, and vertices (MP7). They build on their knowledge of nets learned in previous grades to identify a two-dimensional representation of a three-dimensional figure. They then explore how changing the dimensions of a figure changes the perimeter, area, or volume of the figure (MP8). An understanding of the measures and properties of solids supports students in developing formulas to find volumes of solids and apply these formulas in the next unit.

**Unit 11 Focus Standards**

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

In the following table, **green highlights** indicate major work of the grade, **blue highlights** indicate supporting work, and **yellow highlights** indicate additional work.

<table>
<thead>
<tr>
<th>Standard Text</th>
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</thead>
<tbody>
<tr>
<td>Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</td>
<td>GM:G-MG.A.1</td>
</tr>
<tr>
<td>Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).</td>
<td>GM:G-MG.A.3</td>
</tr>
<tr>
<td>Identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</td>
<td>GM:G-GMD.B.4</td>
</tr>
</tbody>
</table>
### Unit 11 Pacing Guide

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pythagorean Theorem in Three Dimensions</td>
<td>• Identify diagonals and right triangles within cubes.</td>
<td>GM:G-MG.A.1, GM:G-MG.A.3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>• Solve for unknown side lengths of right triangles within a cube.</td>
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</tr>
<tr>
<td>Three-Dimensional Figures and Cross Sections</td>
<td>• Classify a three-dimensional figure and identify the characteristics (base, edge, etc.).</td>
<td>GM:G-GMD.B.4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>• Determine the horizontal and vertical cross sections of three-dimensional figures.</td>
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<tr>
<td></td>
<td>• Determine the three-dimensional figure generated by a rotation of a two-dimensional figure.</td>
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</tr>
<tr>
<td>Solids</td>
<td>• Determine the number of faces, edges, and vertices of a given solid.</td>
<td>GM:G-MG.A.1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>• Classify a solid.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Describe and apply relationships among the faces, edges, and vertices of solids.</td>
<td></td>
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</tr>
<tr>
<td>Drawing Three-Dimensional Figures</td>
<td>• Relate two-dimensional perspective views to three-dimensional figures.</td>
<td>GM:G-MG.A.1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>• Construct a three-dimensional figure, given the top or bottom, side, and front views.</td>
<td></td>
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</tr>
<tr>
<td>Effects of Changing the Dimensions of a</td>
<td>• Identify the effect on other measurements when the dimensions of a shape are changed proportionally.</td>
<td>GM:G-MG.A.1</td>
<td>2</td>
</tr>
<tr>
<td>Figure</td>
<td>• Calculate perimeter, area, or volume when the dimensions of a shape are changed proportionally.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modeling 3-D Figures</td>
<td>• Identify a two-dimensional representation of a three-dimensional figure.</td>
<td>GM:G-MG.A.1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>• Describe a three-dimensional figure that can be created from a two-dimensional representation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Create a model of a three-dimensional figure.</td>
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<tr>
<td>Unit Test</td>
<td></td>
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<td>1</td>
</tr>
</tbody>
</table>

### Discussion Questions & Answers

1. An isosceles trapezoid is rotated around its longer base. Describe the three-dimensional solid that is formed. Describe two different cross sections of the solid that is formed.
   1. The rotation would produce a figure that consists of two cones, one pointed upward and one pointed downward, and a cylinder in between the two cones. The bases of the cones would be the bases of the cylinder.
b. Assuming that the figure is oriented with the points of the cones in a vertical alignment, a horizontal cross section of the figure would be a circle. A vertical cross section through the vertices of the cones would look like an isosceles trapezoid and its reflection across its longer base.

2. A net of a cube can be formed by a two-dimensional arrangement of the six faces. Explain why some arrangements of the faces of a cube create valid nets and some arrangements do not.
   a. The net of a cube uses 6 squares connected along various edges. You need to be able to fold a net into the three-dimensional solid it represents without any faces overlapping. Only a certain number of arrangements of squares will allow you to fold the net into a cube. Other arrangements result in faces overlapping and the cube not being complete.

3. A three-dimensional figure is formed by stacking incrementally smaller squares on top of each other until the last “layer” is a single point. What is the figure that is formed?
   a. Since each square gets incrementally smaller, the figure would taper as it reaches to the single point. This would result in a square pyramid being formed.

4. Explain how a line segment can be used to test if a polyhedron is concave or convex.
   a. Pick any two points on the polyhedron and use a line segment to connect them. If the line segment is on the edge of a polyhedron or inside the polyhedron, it is convex. If the line segment is outside the polyhedron, it is concave.

Common Misconceptions

- Relationship between two-dimensional and three-dimensional geometric figures
  - Students may not understand that three-dimensional geometric figures are formed from projections and rotations of two-dimensional geometric figures.

- Nets of solids
  - Students may incorrectly think that any two-dimensional arrangement of the faces or sides of a three-dimensional figure can be used as a net of the three-dimensional figure.
  - Students may have trouble with the spatial understanding necessary to be able to identify a three-dimensional figure represented by a given two-dimensional net correctly.

- Concave and convex
  - Students may confuse concave and convex when analyzing three-dimensional figures.
  - Students may incorrectly try to apply the Euler formula to a concave polyhedron.

Classroom Challenge

Problem:

1. A solid is sliced to produce a rectangular cross section. Name three possible solids that can have a rectangular cross section. Describe how the cross section was created and create a sketch to go along with your answer.
2. The volume of the solid described above is 100 cubic units. Give possible dimensions for two of the solids you described above and show that the dimensions you have chosen result in solids with the given volume.

3. A cube is sliced from one vertex to the opposite vertex along the diagonal of one of the faces by a plane that is perpendicular to the face. If the volume of the cube is 64 cubic units, what is the area of the figure created by the cross section?

**Possible solution pathway:**

1. Possible answers include but are not limited to the following:
   - a cylinder that is sliced from base to base by a plane that is perpendicular to the base
   - any prism sliced by a plane that is parallel to one of the rectangular faces
   - a rectangular pyramid sliced by a plane that is parallel to the base
2. The answers here will vary greatly based on the solids chosen for the first part of the task. A correct answer should clearly list the dimensions of the solid and show the work necessary to verify that the volume of the solid is 100 cubic units. Here are two examples:

<table>
<thead>
<tr>
<th></th>
<th>cylinder</th>
<th>rectangular prism</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r = 2; h = \frac{25}{\pi} )</td>
<td>( l = 4; w = 5; h = 5 )</td>
</tr>
<tr>
<td></td>
<td>( V = \pi r^2 h )</td>
<td>( V = lwh )</td>
</tr>
<tr>
<td></td>
<td>( V = \pi (2^2) \left( \frac{25}{\pi} \right) )</td>
<td>( V = (4)(5)(5) )</td>
</tr>
<tr>
<td></td>
<td>( V = 100 )</td>
<td>( V = 100 )</td>
</tr>
</tbody>
</table>

3. The cross section that results from this slice is a rectangle with a length that is equal to the length of one side of the cube and a width that is equal to the length of the diagonal of any face.

![Diagram of a cube cut by a plane](image)

First, find the length of one side of the cube using the given area.

\[
A = s^3 \\
64 = s^3 \\
\sqrt[3]{64} = s \\
4 = s
\]

This means the length of the rectangular cross section is 4 units.

Now, find the length of the diagonal of a face. The diagonal forms a right triangle with two sides of the face, so the Pythagorean theorem can be used.

\[
4^2 + 4^2 = c^2 \\
16 + 16 = c^2 \\
32 = c^2 \\
4\sqrt{2} = c
\]

The width of the rectangular cross section is \(4\sqrt{2}\) units.

The area of the rectangular cross section is found by multiplying the length by the width. \((4)(4\sqrt{2}) = 16\sqrt{2}\). The area of the rectangular cross section is \(16\sqrt{2}\) square units.

Teacher notes:
For struggling students, ask them to sketch nets for various solids. Note that any solid that has a rectangle as a part of its net will have at least one way to make a rectangular cross section. Provide students with a list of formulas for the volumes of solids from the unit. Encourage students to guess and check combinations of dimensions to try to find what works to get a volume of 100 cubic units.

For advanced students, ask them to determine more than three ways to get rectangular cross sections from the solids covered in the unit.

**Unit 12: Volume and Surface Area**

*Estimated Unit Time: approx. 12 Class Periods*

In the unit Volume and Surface Area, students build on their knowledge of volume and surface area learned in previous grades. They calculate the volume and surface area of prisms, pyramids, cylinders, cones, and spheres. They also calculate the volume of oblique figures: prisms, pyramids, cylinders, and cones. Using this foundation, students practice calculating surface area and volume of composite figures. Students work backward to determine unknown measurements of geometric figures given volume and surface area. The work completed in this unit supports the major work of using geometric shapes, their measures, and their properties in mathematical and real-world problems.

**Unit 12 Focus Standards**

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

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<td>Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</td>
<td>GM:G-MG.A.1</td>
</tr>
<tr>
<td>Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).</td>
<td>GM:G-MG.A.2</td>
</tr>
<tr>
<td>Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).</td>
<td>GM:G-MG.A.3</td>
</tr>
<tr>
<td>Give an informal argument, e.g., dissection arguments, Cavalieri’s principle, or informal limit arguments, for the formulas for the circumference of a circle; area of a circle; volume of a cylinder, pyramid, and cone.</td>
<td>GM:G-GMD.A.1</td>
</tr>
</tbody>
</table>
## Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

<table>
<thead>
<tr>
<th>Standard Text</th>
<th>Standard ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.</td>
<td>GM:G-GMD.A.3</td>
</tr>
</tbody>
</table>

### Unit 12 Pacing Guide

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
</table>
| **Volume of Prisms** | • Write expressions to represent the volumes or unknown measures of right and oblique prisms.  
• Calculate the volume or an unknown measure of a right prism based on a mathematical or real-world model.  
• Calculate the volume or an unknown measure of an oblique prism based on a mathematical or real-world model. | GM:G-MG.A.1  
GM:G-MG.A.3 | 2 |
| **Volume of Pyramids** | • Write expressions to represent the volumes or unknown measures of right and oblique pyramids.  
• Calculate the volume or an unknown measure of a right pyramid based on a mathematical or real-world model.  
• Calculate the volume or an unknown measure of an oblique pyramid based on a mathematical or real-world model. | GM:G-GMD.A.1  
GM:G-GMD.A.3 | 1 |
| **Volume of Cylinders, Cones, and Spheres** | • Write expressions to represent the volumes or unknown measures of cylinders and cones.  
• Solve mathematical and real-world problems involving the volume of right and oblique cylinders.  
• Solve mathematical and real-world problems involving the volume of right and oblique cones.  
• Solve mathematical and real-world problems involving the volume of spheres. | GM:G-GMD.A.1  
GM:G-GMD.A.3 | 1 |
| **Cavalieri’s Principle and Volume of Composite Figures** | • Write an expression to represent the volume of a composite figure.  
• Calculate the volumes of composite figures, including those that model real-world objects. | GM:G-MG.A.2  
GM:G-GMD.A.3 | 1 |
| **Surface Area of a Cone** | • Determine the base area and lateral area of a cone.  
• Calculate the surface area of a cone. | GM:G-MG.A.1 | 2 |
Lesson | Objectives | Standards | Number of Days
--- | --- | --- | ---
Surface Area | • Solve mathematical and real-world problems involving lateral area of prisms, cylinders, pyramids, and cones.  
• Solve mathematical and real-world problems involving surface area of prisms, cylinders, cones, spheres, and pyramids.  
• Solve mathematical and real-world problems about lateral and surface areas of composite figures. | GM.G-MG.A.1 | 2
Spheres | • Apply the formulas for volume and surface area of a sphere to determine unknown measurements. | GM.G-MG.A.3 | 2
Unit Test |  |  | 1

Discussion Questions & Answers

1. The radius of a sphere is doubled in length. Explain the effect on the surface area and the volume of the sphere.
   a. The formula for the surface area of a sphere involves squaring the radius. Doubling the radius means multiplying it by 2. Since \(2^2\) is 4, the surface area of the new sphere would be 4 times the original surface area.
   b. The formula for the volume of a sphere involves cubing the radius. Doubling the radius means multiplying it by 2. Since \(2^3\) is 8, the volume of the new sphere would be 8 times the original volume.

2. Explain how to find the side length of a cube given the surface area.
   a. A cube is made up of 6 congruent square faces. The formula for the surface area of a cube is \(SA = 6s^2\), where \(s\) is the length of a side. If you are given the surface area, you can rewrite the formula to find \(s\) in terms of \(SA\), \(s = \sqrt{\frac{SA}{6}}\). So, given a value for the surface area, you can divide by 6, then find the square root of the result to find the length of a side, \(s\).

3. Compare and contrast the top, front, and side views of a right cylinder, a right cone, and a sphere. Describe how viewing a sphere from different perspectives is like viewing a cube from different perspectives.
   a. A cylinder looks like a rectangle from the front and the side. It looks like a circle from the top. A right cone also looks like a circle, but from the bottom (with a dot for the vertex in the center); it looks like a triangle from the front and the side. A sphere looks like a circle no matter from which perspective—top, front, or side.
b. A cube looks like a square from the top, front, or side. This is similar to the way that a sphere looks like a circle from the top, front, or side.

Common Misconceptions

- Surface area
  - Students may incorrectly think that surface area is always the same no matter what the volume.
  - Students may have trouble with understanding surface area as a two-dimensional concept and volume as a three-dimensional concept.

- Volume
  - Students may not understand that volume is a three-dimensional projection of two-dimensional area.
  - Students may not make the connection between tiling for area and filling with unit cubes for volume.
  - Students may not understand how doubling the dimensions of a solid results in a volume that is 8 times the original volume.
  - Student may incorrectly think that volume is always the same no matter what the surface area.
  - Students may not understand that volume and capacity are related and are often the same amount.

- Formulas
  - Students might not be sure about when to apply a surface area formula and when to apply a volume formula if it is not explicitly specified in the problem.

Classroom Challenge

An architect is designing a building composed of a square prism with a hemisphere for a dome on top. During one of the changes to the drawings, the architect lost the dimensions to the pieces of the building. The architect found some earlier notes that indicate the building height—from the ground to the top of the dome—is twice the length of the building’s base. According to the notes to the heating and air conditioning contractor, the total volume of the building to the nearest cubic foot is 47,565 ft³. What are the dimensions of the prism and the radius of the hemisphere?
Possible solution pathway:

The diagram shows the given dimensions. Since the prism has a square base, the dimensions of the base are \( x \). The diameter of the hemisphere is the length of the side of the base of the prism or \( x \). So, the radius of the hemisphere is \( \frac{x}{2} \). The total height of the building is the sum of the radius of the hemisphere and the height of the prism.

Total height: \( 2x = h + \frac{x}{2} \) or \( h = 2x - \frac{x}{2} \).

The volume of the building is equal to the sum of its parts.

\[
V_{\text{building}} = V_{\text{hemisphere}} + V_{\text{prism}}
\]

First, find the expression for the volume of the hemisphere in terms of \( x \).

\[
V_{\text{hemisphere}} = \frac{1}{2} \left( \frac{4}{3} \pi r^3 \right) = \frac{1}{2} \left( \frac{4}{3} \pi \left( \frac{x}{2} \right)^3 \right) = \frac{1}{2} \left( \frac{4}{3} \pi \left( \frac{x^3}{8} \right) \right) = \frac{1}{2} \left( \frac{\pi x^3}{6} \right) = \frac{\pi x^3}{12}
\]

Then find the expression for the volume of the prism in terms of \( x \).

\[
V_{\text{prism}} = (x)(x)(h) = x^2 \left( 2x - \frac{x}{2} \right) = \frac{3x^3}{2}
\]

Use these expressions and the given volume to write an equation in terms of \( x \).

\[
47,565 = \frac{\pi x^3}{12} + \frac{3x^3}{2}
\]

Solve. This work shows one path to \( x \). Note that there are several paths that depend on when 3.14 is substituted for \( \pi \) and how results are rounded along the way. Ideally, all such paths lead to the same solution.

\[
47,565 = x^3 \left( \frac{\pi}{12} + \frac{3}{2} \right)
\]

\[
47,565 = x^3 \left( \frac{\pi + 18}{12} \right)
\]
\[
\frac{570,780}{\pi + 18} = x^3 \\
\frac{26,998}{30} \approx x^3
\]

The dimensions of the prism are 30 feet, 30 feet, and 45 feet. The radius of the hemisphere is 15 feet.

Teacher notes:
For visual learners, remind them to draw a sketch labeling the given parts. Have students write the formulas and practice substituting values for \( x \) and finding the total volume. This will show them the steps in the normal order, allowing them to envision the reverse steps for finding the value of \( x \) with the given volume.

For advanced learners, ask them to write a hypothesis about the change in dimensions if the volume doubles. What relationships stay the same? Why?

UNIT 13: APPLICATIONS OF PROBABILITY

Estimated Unit Time: approx. 12 Class Periods

In the unit Applications of Probability, students build on their knowledge of probability learned in previous grade levels. Students learn to describe and represent outcomes and explore how to use these outcomes to determine theoretical and experimental probability. Students learn to represent sample spaces using permutations and combinations. They identify mutually exclusive and independent events and calculate related probabilities using addition and multiplication rules. Students calculate conditional probabilities using formulas, Venn diagrams, and two-way tables, often using these calculations to determine if events are independent. Students complete a performance task, in which they apply their probability skills in a decision-making context.

Unit 13 Focus Standards

The following focus standards are intended to guide teachers to be purposeful and strategic in both what to include and what to exclude when teaching this unit. Although each unit emphasizes certain standards, students are exposed to a number of key ideas in each unit, and as with every rich classroom learning experience, these standards are revisited throughout the course to ensure that students master the concepts with an ever-increasing level of rigor.

In the following table, green highlights indicate major work of the grade, blue highlights indicate supporting work, and yellow highlights indicate additional work.

<table>
<thead>
<tr>
<th>Standard Text</th>
<th>Standard ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prove that all circles are similar.</td>
<td>GM:G-C.A.1</td>
</tr>
</tbody>
</table>
### Standard Text

<table>
<thead>
<tr>
<th>Standard Text</th>
<th>Standard ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</td>
<td>GM:S-CP.A.1</td>
</tr>
<tr>
<td>Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</td>
<td>GM:S-CP.A.2</td>
</tr>
<tr>
<td>Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.</td>
<td>GM:S-CP.A.3</td>
</tr>
<tr>
<td>Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</td>
<td>GM:S-CP.A.4</td>
</tr>
<tr>
<td>Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</td>
<td>GM:S-CP.A.5</td>
</tr>
<tr>
<td>Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.</td>
<td>GM:S-CP.B.6</td>
</tr>
<tr>
<td>Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.</td>
<td>GM:S-CP.B.7</td>
</tr>
</tbody>
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### Standard ID

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<td>GM:S-CP.A.2</td>
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<td>GM:S-CP.A.3</td>
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<td>GM:S-CP.A.4</td>
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<td>GM:S-CP.A.5</td>
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<td>GM:S-CP.B.6</td>
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<td>GM:S-CP.B.7</td>
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### Unit 13 Pacing Guide

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
</table>
| Sets and Venn Diagrams | • Identify and represent elements of sets and subsets, including the empty and universal sets.  
• Represent and interpret the union and intersection of sets using set notation and Venn diagrams.                                           | GM:S-CP.A.1 | 1              |

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### Lesson Objectives

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Objectives</th>
<th>Standards</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finding Outcomes</td>
<td>• Identify possible outcomes for an event.</td>
<td>GM:S-CP.A.1</td>
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<tr>
<td></td>
<td>• Evaluate expressions involving factorials.</td>
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<td></td>
<td>• Solve combination problems, including finding a subset of the total number of possible combinations.</td>
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</tr>
<tr>
<td></td>
<td>• Solve permutation problems, including finding a subset of the total number of possible permutations.</td>
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<td></td>
</tr>
<tr>
<td>Theoretical and Experimental Probability</td>
<td>• Identify the sample space of an experiment and the complement of an event.</td>
<td>GM:S-CP.A.1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>• Calculate theoretical and experimental probability.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Independent and Mutually Exclusive Events</td>
<td>• Identify mutually exclusive and independent events.</td>
<td>GM:S-CP.A.2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>• Calculate probabilities using the addition rule.</td>
<td>GM:S-CP.B.7</td>
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</tr>
<tr>
<td></td>
<td>• Calculate probabilities using the multiplication rule of independent events.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditional Probability</td>
<td>• Use calculations to determine if two events are independent.</td>
<td>GM:S-CP.A.3</td>
<td>2</td>
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<tr>
<td></td>
<td>• Calculate conditional probabilities using formulas and Venn diagrams.</td>
<td>GM:S-CP.A.5</td>
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<tr>
<td></td>
<td>• Calculate probabilities of compound events.</td>
<td>GM:S-CP.B.6</td>
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<tr>
<td>Probability and Two-Way Tables</td>
<td>• Construct a two-way table.</td>
<td>GM:S-CP.A.4</td>
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</tr>
<tr>
<td></td>
<td>• Use a two-way table to determine if two events are independent.</td>
<td>GM:S-CP.A.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Compute conditional probabilities from data displayed in a two-way table.</td>
<td>GM:S-CP.B.6</td>
<td></td>
</tr>
<tr>
<td>Performance Task: Geometric Probability Models</td>
<td>•</td>
<td>GM:S-CP.A.1</td>
<td>2</td>
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<tr>
<td>Unit Test</td>
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<td>1</td>
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</table>

### Discussion Questions & Answers

1. A fair coin is flipped 10 times. The result is 3 heads and 7 tails. Compare the experimental probability with the theoretical probability. If the experiment is repeated 100 times, how should the overall results compare with the theoretical probability? Explain.
   
a. The experimental probability in this case is 0.7 for tails and 0.3 for heads. Since the coin is fair, the theoretical probability would be 0.5 for tails or heads.

   b. If the experiment is repeated 100 times, the results each time would vary. However, the overall number of heads and tails should converge to the same number for each according to the law of large numbers. So, the experimental probability over repeated experiments would get closer and closer to 0.5 for tails or heads.
2. A spinner has three sections that are yellow, blue, and green. After 100 spins, the results are 65 yellow, 3 blue, and 32 green. Based on the experimental probability, what are the likely relative sizes of the sections on the spinner? Explain why.
   a. Because the yellow section has the most results, it is likely that the yellow section is the largest. Since the green section has fewer results than the yellow section but more than the blue section, it is likely smaller than the yellow section and larger than the blue section. Since the blue section has the fewest results, it is likely the smallest of the sections.

3. Explain the difference between independent and dependent events. Give examples of each.
   a. Events are independent if the probability of one event does not have any effect on the probability of the second event. Events are dependent if the probability of a second event depends on the probability of a first event.
   b. An example of independent events are a flip of a coin and a spin of a spinner. The probabilities of a certain result for the coin flip is not dependent on the probability of a certain result for the spin of the spinner. An example of dependent events is randomly drawing a certain color of marble from a bag and not replacing it, then randomly drawing another marble from the bag. Since the first marble was removed, it affects the probability for the second drawing.

Common Misconceptions

- Experimental and theoretical probability
  - Students may not understand that probability ranges from 0 to 1, or 0% to 100%.
  - Students may confuse experimental and theoretical probability.
  - Students may not be convinced that with repeated experimentation with equally likely events, experimental probability will converge on theoretical probability.

- Outcomes and events
  - Students may make errors in counting all possible outcomes for a given probability scenario.
  - Students may incorrectly attempt to assign equally likely probabilities to events that are not equally likely.
  - Students may incorrectly identify events as independent or dependent.
  - Students may incorrectly think that an independent event is dependent on an immediately preceding outcome.

- Sample sizes
  - Students may not understand that the greater the sample size, the more valid it is to generalize from the data.

Classroom Challenge
The two-way table shows some of the results of a survey of members of a basketball fan site about the television coverage of a tournament. Results were collected from 2,573 respondents.

<table>
<thead>
<tr>
<th></th>
<th>Watched Championship Game</th>
<th>Did Not Watch Championship Game</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watched Semifinal Games</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Did Not Watch Semifinal Games</td>
<td>845</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. If the probability of randomly selecting a fan that watched both the semifinal games and the championship is half the probability of selecting a fan who did not watch either of the games, complete the table with possible values.

2. Using the completed table, find the probability to the nearest hundredth of randomly selecting a fan who watched the semifinal games and did not watch the championship. Then find the probability to the nearest hundredth of randomly selecting a fan who did not watch the semifinal games.

Possible solution pathway:

1. In both probabilities, the number of possible outcomes is the number of respondents, 2,573. For the probability to be half as much, the larger probability has a numerator double the smaller.
   a. Pick any number for the blue-shaded square, 194.
   b. Find the total number of fans who watched the championship game, (194 + 845).
   c. Find the number of fans who did not watch the championship game, (2,573 – 1,039).
   d. Double the number in the blue-shaded square for the gray-shaded square, (2)(194).
   e. Find the number of fans who watched the semifinal games but did not watch the championship game, (1,534 – 388).
   f. Find the sums to complete the total column, (194 + 1,146) and (845 + 388).
g. Confirm the sum of the total column matches the total respondents, \((1,340 + 1,233)\).

<table>
<thead>
<tr>
<th>Watched Championship Game</th>
<th>Did Not Watch Championship Game</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watched Semifinal Games</td>
<td>194</td>
<td>1,146</td>
</tr>
<tr>
<td>Did Not Watch Semifinal Games</td>
<td>845</td>
<td>388</td>
</tr>
<tr>
<td>Total</td>
<td>1,039</td>
<td>1,534</td>
</tr>
</tbody>
</table>

2. The probability of randomly selecting a fan who watched the semifinal games but did not watch the championship game is 0.45. The probability of selecting a fan who did not watch the semifinal games is 0.52.

Teacher notes:

Struggling students are likely unsure which number to pick. Encourage some trial and error. Ask them to think of a reasonable number once they have determined that one is double the other. For students who are still hesitant, ask them to complete a chart with much smaller numbers and then apply the same principles to the given information.

For advanced students, ask them how many more numbers need to be given so that all answers are the same. What is the minimum number of data cells that should be provided so that all students get the same answer?

TIPS ON EFFECTIVE DISCUSSIONS

Edgenuity courseware supports students in using mathematical language precisely (MP6) and in constructing viable arguments to justify their reasoning (MP3) by including discussion questions for students to complete in a classroom or virtual discussion room setting.

- Make expectations clear. How many times should students post in discussions? Are they required to respond to all questions or just one in each unit? You may also wish to require that students respond to at least one other student’s post in addition to answering the original question themselves.
• Share appropriate rules for online discussions with students. Remind them that online discussions don’t convey tone as well as face-to-face discussions, and they should be careful to write things that cannot be misinterpreted (e.g., avoid sarcasm). Likewise, let students know that they should never post anything in a discussion forum that they would not say in a face-to-face discussion.

• Encourage students to ask follow-up questions in the forum. Discuss with students what makes an effective follow-up question. For example, questions should elicit elaboration, as opposed to having a single correct answer.

• Let students know that there is no shame in changing one’s views in response to new information posted by others. In fact, that is part of what discussions are all about.

• If you facilitate online discussions, post discussion questions at the start of the unit so students can return to them multiple times. Students are often motivated to contribute more when they see the contributions of other students over time. Once you have posted questions, send students an email to let them know that forums are open.

• If you facilitate face-to-face discussions, make clear to students what they should come prepared to discuss. You may wish to provide the discussion questions in advance of the discussion. When discussion is happening in real time, this allows less confident students to feel more prepared and have evidence to support their ideas.
### Course Vocabulary List

All of the words below are taught to students in the course of instruction. Students have access to definitions in their lesson glossaries and can look up any word at any time within instruction and assignments.

<table>
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<th>Vocabulary</th>
<th>Definition</th>
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<td>acute angle</td>
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<td>acute triangle</td>
<td>center of dilation</td>
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<td>center of rotation</td>
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<td>alternate exterior angles</td>
<td>central angle</td>
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<table>
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<th>Term</th>
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Edgenuity Algebra I Interactive Tools

Students use a variety of powerful interactive instructional tools to help them build content knowledge and essential skills, support them in learning procedures, and facilitate the exploration of new or challenging concepts throughout Edgenuity Geometry.

Unit 1

Measuring Length and Angles
- An interactive ruler allows students to determine the length of a line segment.
- An interactive protractor allows students to determine the measure of an angle.

Introduction to Proof
- Interactive tiles allow students to complete a two-column proof.

Linear Pairs and Vertical Angles
- Interactive tiles allow students to complete a two-column proof.

Complementary and Supplementary Angles
- Interactive tiles allow students to complete a two-column proof.

Unit 2

Reflections
- An interactive diagram allows students to explore the properties of reflections.
- An interactive protractor allows students to determine angle measures in a pre-image and image.
- An interactive ruler allows students to determine side lengths in a pre-image and image.
- An interactive graph allows students to explore reflections on the coordinate plane.

Translations
- An interactive diagram allows students to explore the properties of translations.
- An interactive graph allows students to explore translations on the coordinate plane.
Rotations
- An interactive diagram allows students to explore the properties of rotations.
- An interactive protractor allows students to measure angles in a pre-image and image.
- An interactive ruler allows students to measure side lengths in a pre-image and image.

Symmetry
- An interactive diagram allows students to explore possible lines of symmetry in a figure.

UNIT 3

Parallel and Perpendicular Lines
- An interactive compass tool allows students to construct parallel lines through a given point.

Proving Lines Parallel
- Interactive tiles allow students to complete a two-column proof.

Slopes of Parallel and Perpendicular Lines
- An interactive graph allows students to determine the slopes of parallel and perpendicular lines on the coordinate plane.

Writing Linear Equations
- An interactive graph allows students to construct a parallel line through a given point on the coordinate plane.

UNIT 4

Constructing and Analyzing Triangles
- An interactive protractor allows students to determine the measure of an angle.
- An interactive diagram allows students to explore the properties of different triangles.

Triangle Angle Theorems
- Interactive tiles allow students to complete a two-column proof.

Triangles and Their Side Lengths
- An interactive diagram allows students to explore forming triangles with different combinations of side lengths.
- An interactive compass tool allows students to construct an equilateral triangle.

Triangle Inequalities
- An interactive diagram allows students to explore relationships between angle measures and side lengths in a triangle.

Isosceles Triangles
- An interactive diagram allows students to explore the properties of an isosceles triangle.

Centroid and Orthocenter
- An interactive diagram allows students to explore the location of the orthocenter of different triangles.
Incenter and Circumcenter
- An interactive diagram allows students to explore angle bisectors of different triangles.
- An interactive compass tool allows students to find the incenter and circumcenter of different triangles.

Construct Regular Polygons
- An interactive compass tool allows students to construct different regular polygons inscribed in a circle.

UNIT 5

Triangle Congruence: SAS
- Interactive tiles allow students to complete a two-column proof.

Using Triangle Congruence Theorems
- Interactive tiles allow students to complete a two-column proof.

UNIT 6

Dilations
- An interactive diagram allows students to explore the properties of dilations and scale factors.
- An interactive ruler allows students to measure lengths and determine the scale factor of a dilation.

Similar Figures
- An interactive graph allows students to observe the properties of dilations on the coordinate plane.
- An interactive ruler allows students to measure lengths and determine the scale factor of a dilation on the coordinate plane.

Triangle Similarity: AA
- Interactive tiles allow students to complete a two-column proof.

UNIT 7

Special Right Triangles
- An interactive diagram allows students to explore the 45°-45°-90° triangle theorem.
- An interactive ruler allows students to measure side lengths in an isosceles right triangle.

Trigonometric Ratios
- Interactive tiles allow students to complete a table by labeling the sides of a right triangle.
UNIT 8

Classifying Quadrilaterals
- An interactive diagram allows students to explore the sums of angle measures in quadrilaterals.

Parallelograms
- Interactive tiles allow students to complete a two-column proof.
- An interactive diagram allows students to explore diagonals of a parallelogram.

Proving a Quadrilateral Is a Parallelogram
- Interactive tiles allow students to complete a two-column proof.

Special Parallelograms
- Interactive tiles allow students to complete a two-column proof.
- An interactive diagram allows students to explore the properties of special quadrilaterals.

Trapezoids and Kites
- Interactive tiles allow students to complete a two-column proof and sort the properties of trapezoids and kites.

UNIT 9

Introduction to Circles
- Interactive tiles allow students to label parts of a circle and complete a two-column proof.

Central Angles
- An interactive diagram allows students to explore the relationship between a radius and a tangent.
- An interactive protractor allows students to measure angles formed by the intersection of a radius and a tangent.

Inscribed Angles
- Interactive diagrams allow students to explore the properties of inscribed angles in different figures.

Secants, Tangents, and Angles
- An interactive diagram allows students to explore the properties of angles formed by intersecting chords.
- An interactive diagram allows students to explore the properties of angles and arcs formed by intersecting secants.

Circumference and Arc Length
- Interactive diagrams allow students to explore radian measure in different figures.

Area of a Circle and a Sector
- An interactive diagram allows students to explore the area of sectors of circles.

Angle Relationships
• Interactive diagrams allow students to explore angles and arcs of different circles.
• An interactive diagram allows students to explore polygons that can be created by radii and chords.

Performance Task: Circle Constructions
• An interactive compass tool allows students to complete a variety of constructions to meet a set of given criteria.

UNIT 10

N/A

UNIT 11

Modeling 3-D Figures
• An interactive diagram allows students to explore an isometric projection of a rectangular prism.

UNIT 12

N/A

UNIT 13

Sets and Venn Diagrams
• Interactive tiles allow students to complete a Venn diagram.

Theoretical and Experimental Probability
• An interactive coin toss simulation allows students to perform a probability experiment.

Probability and Two-Way Tables
• Interactive tiles allow students to create a two-way table from a given scenario.

COURSE CUSTOMIZATION

Edgenuity is pleased to provide an extensive course customization toolset, which allows permissioned educators and district administrators to create truly customized courses that ensure that our courses can meet the demands of the most rigorous classroom or provide targeted assistance for struggling students.

Edgenuity allows teachers to add additional content two ways:

1. Create a brand-new course: Using an existing course as a template, you can remove content, add lessons from the Edgenuity lesson library, create your own activities, and reorder units, lessons, and activities.
2. Customize a course for an individual student: Change an individual enrollment to remove content, add lessons, add individualized activities, and reorder units, lessons, and activities.

Below you will find a quick-start guide for adding lessons in from a different course or from our lesson library.

In addition to adding lessons from another course or from our lesson library, Edgenuity teachers can insert their own custom writing prompts, activities, and projects.
How do I Create Project or Writing Prompt Activities?

Navigate to where you want to add the new activity, and select the lesson by clicking on the lesson title.

Click the Add Activity button, then select Writing Prompt or Project.

If you previously created new activities, they will display here. Click the activity name to preview the activity instructions.

Click the green plus sign to insert an activity into the lesson.

The activity will be inserted at the top of the unit. You can move the activity to another location in the lesson.

If you are creating a new Writing Prompt, specify the name, description, prompt, grade weight category, and optionally, keywords for scoring, sample answer, and scoring guidance.

If you are creating a new Project, specify the name, description, type, and grade weight category, and provide student resources by entering hyperlinks to web sites or uploading files.

NOTES

- Accepted file types are: .ppt, .pptx, .xls, .xlsx, .doc, .docx, .zip, .pdf, .accdb, .msg.

- Links you create won’t go through the Edgenuity Emissary (Proxy). This means that your IT department will need to ensure that the link is whitelisted or otherwise allowed to be accessed. It also means that items blocked by the Edgenuity proxy may be visible on the sites you link to. In addition, the Edgenuity tools to highlight, translate, read aloud, or add a sticky note will not be present on the site you link to.

- You add activities to courses with no enrollments or on individual student’s courses. It is not possible to add activities to in-flight courses that have enrollments.